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# White Paper

# Volatility: Instruments and Strategies

Clemens H. Glaffig
*Panathea Capital Partners GmbH & Co. KG, Freiburg, Germany*

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# Volatility: Instruments and Strategies

Clemens H. Glaffig
*Panathea Capital Partners GmbH & Co. KG, Freiburg, Germany*

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*This white paper describes the available universe of instruments related to the VIX Index, the gauge for short term implied volatility of options on the S&P500 index and some of the strategies used within this universe. The breadth of the available instruments on VIX is at this stage unmatched by any other Volatility index, but the technical issues described here in the context of VIX related products can also be applied to other indices, should they expand their own universe of derivative instruments.*

## 0. Introduction

Volatility as an expression of fluctuations of the price path of some given market is for many market participants an abstract notion. It is an unwanted market phenomenon, that provides uncertainty and forces them to scale and risk manage their investements. With the rise of financial derivatives for which volatility is one of the major price determinants, it became desirable to adaquately measure, define and model volatility. Market obeservables were directly translated into Volatility indices to have a tractable gauge of market fluctuations. Simply creating an index does not itself create a tradable market however, but with the introduction of Futures, Options and ETPs related to volatility indices, volatility emerged into an asset class in its own right, easily accessible and offering risk-reward combinations not replicable by other markets. Implied volatility can now be traded directly and not only indirectly via Options.
Volatility indices exist, amongst others, on the SPX, DJIA, Nasdaq-100, Russell2000, QQQ, €Stoxx50, DAX, SMI, Gold, Crude Oil, EURUSD, VIX (VVIX), VSTOXX (V-VSTOXX) and Government Bonds (MOVE).

This white-paper focuses on the VIX Index, a gauge of implied volatility of exchange traded options on the S&P500 index ("SPX") and illustrates the available universe of instruments around this index as well as some of the strategies that can be employed within this universe. Amongst the various volatility indices available, it has the largest and most liquid universe of tradable instruments related to it and offers the best basis to enact strategies to unlock new and alternative risk factors and premia.

# 1. The VIX Index
## 1.1 General Comments and Performance

The VIX Index is often portrayed as the “fear index” by the financial media. Indeed, as a measure of implied volatility of traded options of an equity index, it rises with falling equity prices and realizes its sharpest peaks during sharp equity declines, reflecting the rapid rise in investor anxiety, hedging needs and trading activity. Market turbulences lead investors to pay up for hedges via options and consequently drive up implied volatility. Calculating an index reflecting investors’ anxiety and fear gives the investor a quantifiable and market-derived measure of an otherwise abstract concept and at the same time presents a verifiable calculation of the degree of market turbulence for a more formal approach to trading and risk management.

The VIX Index reflects the implied volatility paid for exchange traded SPX options with a life of 30 days. The calculation of the VIX Index appears to be rather complicated in comparison with, e.g., equity indices, which are a weighted sum of its member stocks, the weights reflecting the relative market capitalization or other, fairly intuitive weighting scheme. As detailed further below, for the VIX, it is a weighted sum of options of the first two monthly expirations, with the option-weights proportional to the inverse of the squared strike prices and the expiration months weighted to result in a 30 day average life – not exactly intuitive first hand, it needs to be derived to be appreciated (see Section 1.2)

Looking at the performance graph of the VIX, it seems obvious, that it is a mean reverting index, always drifting towards some low level “equilibrium” or saturated state, with relatively short spikes during market turbulences. Specifically, since the crisis of 2008, when the VIX reached levels around 80%, the VIX has, with a notable number of intermediate spikes, always looked to reach its minimum fluctuation level of around 10% (see below).The correlation between VIX and SPX is negative.

**What does it mean to have a VIX at 20%:** While the path of the VIX reflects the level of implied volatility paid for exchange traded SPX options it still leaves the question, what is really expressed by the level of the VIX. Option prices are generally not very informative, as they change with strike level and time to expiration. A more robust measure for the value of an option is to consider the implied volatility with which the option is priced and traded. The higher the implied volatility, the more costly the option. When the implied volatility is at 20%, it means that for pricing purposes, it is assumed, that the underlying will fluctuate within a range of about $\pm 5{,}75\%$ for the next 30 days.

**A nerdy side note:** The standard way to measure volatility is to calculate the average of the squared and de-meaned index-returns observed at regular spaced intervals over a given time period and annualized. For periods smaller than a day, the de-meaning is often neglected. Another way to express and measure volatility is price duration, i.e. measuring the time it takes for the index to change by a specified, fixed percentage. This way, the squared returns are all of the same quantity; the observations are undertaken at irregular spaced intervals – the “event” times. This approach is particularly of relevance at high frequency. If the underlying index would follow a GBM with constant volatility equal to the VIX Index level, the expected waiting time for the index-return to move by $\pm\delta\%$ is $\frac{\delta^2}{\sigma^2} = \frac{\delta^2}{VIX^2}$ . Viewed this way, the level of the VIX Index is also a statement about expected waiting times for percentage-changes in the SPX.

Following below is a performance graph for the SPX and the VIX from 2007 until the summer of 2019. The correlation between VIX and SPX over this period stands at -72%, the 30day rolling correlation fluctuates mainly between -80% and -90%, see Figures 1.1 & 1.2.

Fig. 1.1:

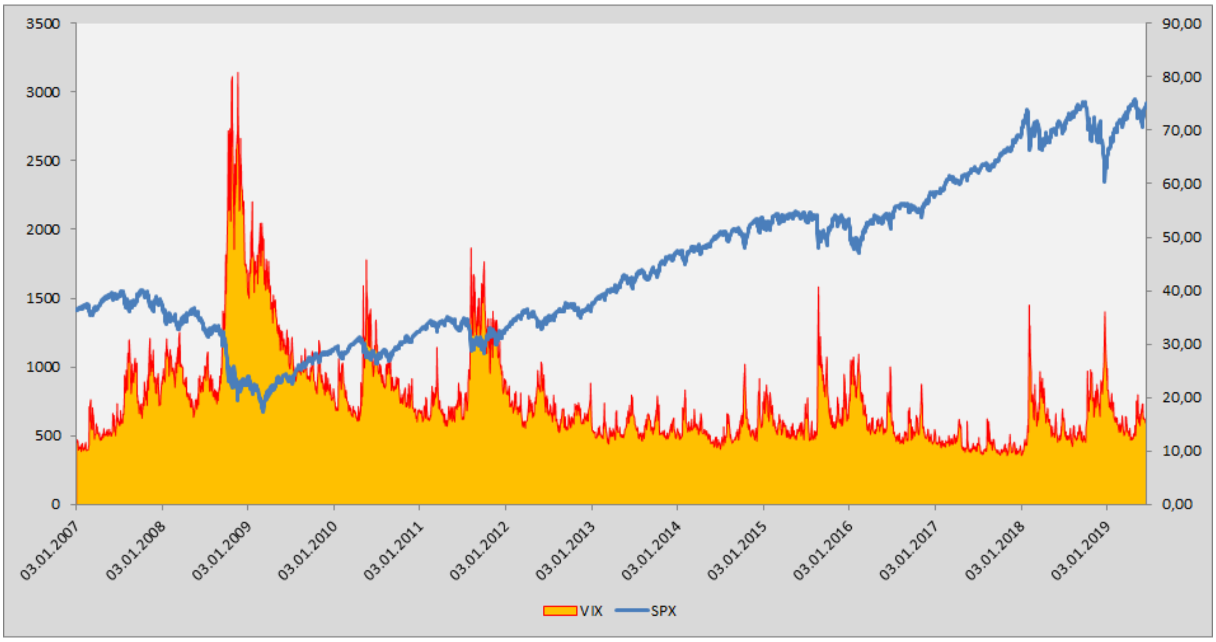
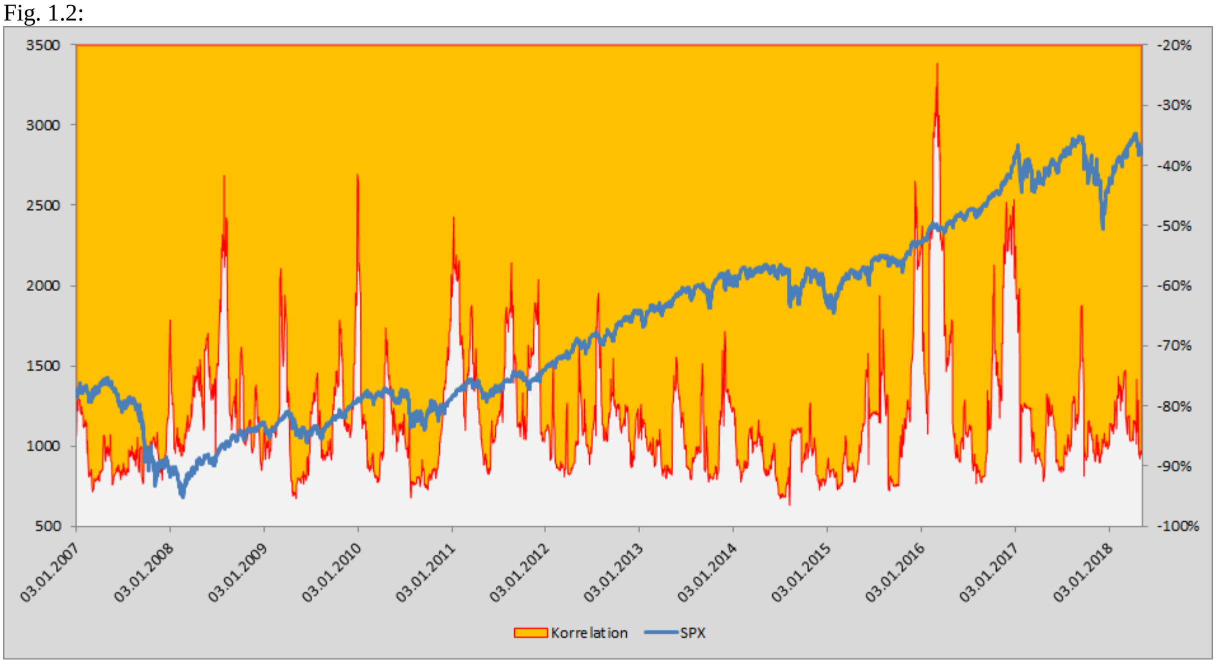
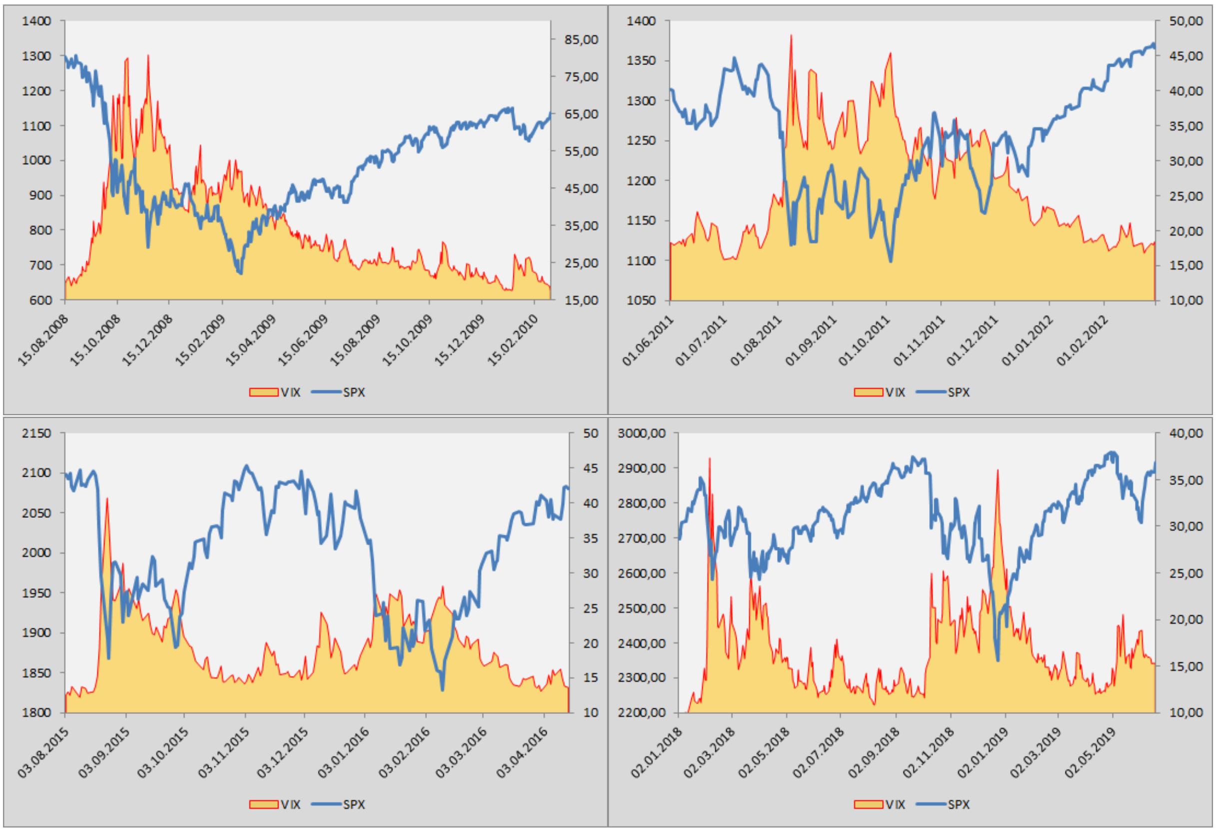



Fig. 1.2:




The following graphs zoom out the periods between Aug2008 – Mar2010, Jun2011 – Mar2012, Aug2015 – Apr2016 and Jan2018 – Jun2019 respectively. The correlations between VIX and SPX over those periods are -80%, -86%, -87% and -77% respectively.

Fig.: 1.3




In these graphs, it can be observed that if the SPX has two consecutive bottoms, with the second at least as low as the first. The VIX has two corresponding spikes, the second at most as pronounced than the first. It seems that the VIX might not only reflect fear, but also negative surprise or degree of "wrong-footedness".

## 1.2 The Calculation of the VIX Index

The VIX Index is supposed to reflect the current priced implied volatility of options on the S&P 500 index with a weighted averaged time to expiration of 30 days. The VIX Index was first introduced in 1993 by the CBOE as a measure of 30-day volatility of ATM S&P 100 options. Originally, the index was calculated from options around the current spot as a true volatility index (using 8 options: Two strikes around spot, two expirees, Puts and Calls). In 2003, the methodology to calculate the VIX changed from being based on S&P 100 Options to S&P 500 Options and from being priced as a volatility index to being priced like a variance swap. Contrary to a volatility swap, a variance swap can be statically hedged. Moreover, it appears to be the true measure of fluctuations. A variance swap takes all strikes of traded options into account (the old VIX considered only 2 strikes) and the payoff of a delta hedged option, which eliminates linear underlying risk and focuses on the volatility premium depends on the $-Gamma weighted realized variance.

To calculate its value, S&P 500 options with two expiration dates are considered: One expiration with less than 30 days and closest to 30 days, the other with expiration above 30 days and closest to 30 days. The options include weekly and monthly SPX Options.

Specifically, for a given expiration T of options on the S&P 500, the implied variance for that expiration is calculated by (see CBOE (2019))

$$\sigma_T^2 = \tfrac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \tfrac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad (0)$$

T time to expiration

F Forward index level derive from option prices

$K_0$ First strike below forward index level F

$K_i$ strike of the i-th out-of –the –money option; a call if $K_0 < K_i$ and a put if $K_0 > K_i$; both put and call if $K_0 = K_i$

$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$

R Risk free interest rate

$Q(K_i)$ The midpoint of the bid-ask spread for each option with strike $K_i$

The VIX Index is then the square root (transferring variance back to the corresponding standard deviation/volatility) of the weighted average of the variance of the two expiration dates bracketing 30 days, resulting in a weighted average life of 30 days:

$$VIX = 100 * \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right\}\frac{N_{365}}{N_{30}}} \qquad (1)$$

$N_{T_1}$= number of minutes to settlement of the near term option

$N_{T_2}$= number of minutes to settlement of the next term option

$N_n$ = number of minutes within n days.

### 1.3 Mathematical formalism: How to derive the VIX valuation formula

For a twice differentiable function g (also in a functional derivative sense), it is known that the risk-neutral expected payoff at time T of $g(S_T)$ for some underlying index S (with 0 dividend yield) is given by

$$\mathbb{E}[g(S_T)] = g(F) + e^{rT}\int_0^F g''(K)P(K)dK + e^{rT}\int_F^\infty g''(K)C(K)dK \qquad (2)$$

with P and C denoting the Put and Call prices with strike K and underlying S, F the T-time forward of S and the expectation is w.r.t. to the risk neutral measure (see, e.g., J.Gatheral (2006)).

Considering $\text{g}(S_T) = \log({}^{S_T}/_F)$ , the above formula delivers the value of the so called log-contract:

$$\mathbb{E}[log(S_T/F)] = -e^{rT}\int_0^F \frac{P(K)}{K^2}dK - e^{rT}\int_F^\infty \frac{C(K)}{K^2}dK$$

On the other hand, employing Ito's lemma and assuming deterministic rates, one obtains for the discounted price process $\hat{S}_t := e^{-rt}S_t$, $\widehat{S}_0 = S_0 = e^{-rT}F$ , $\hat{\sigma}_t^2 = \sigma_t^2$ and

$$log(\hat{S}_T/\hat{S}_0) = log(S_T/F) = \int_0^T dlog(\hat{S}_t) = \int_0^T \frac{d\hat{S}_t}{\hat{S}_t} - \frac{1}{2}\int_0^T \sigma_t^2\, dt$$

Taking expectation w. r. to the risk neutral measure, using the fact that $\hat{S}_t$ is a martingale under that measure and hence $\mathbb{E}\left[\int_0^T \frac{d\hat{S}_t}{\hat{S}_t}\right] = 0$ , one obtains

$$\sigma^2 = -\frac{2}{T}\mathbb{E}[log(S_T/F)] = \frac{2}{T}e^{rT}\int_0^F \frac{P(K)}{K^2}dK + \frac{2}{T}e^{rT}\int_F^\infty \frac{C(K)}{K^2}dK$$

To get from here to the calculation of the VIX Index, the (constant) variance of a given maturity can be expressed as

$$\begin{aligned}
\sigma^2\frac{T}{2} = e^{rT}\int_0^F \frac{P(K)}{K^2}dK + e^{rT}\int_F^\infty \frac{C(K)}{K^2}dK \\
= e^{rT}\int_0^{K_0} \frac{P(K)}{K^2}dK + e^{rT}\int_{K_0}^\infty \frac{C(K)}{K^2}dK + e^{rT}\int_{K_0}^F \frac{(P(K)-C(K))}{K^2}dK \\
= e^{rT}\int_0^\infty \frac{Q(K)}{K^2}dK + \int_{K_0}^F \frac{(K-F)}{K^2}dK \cong e^{rT}\int_0^\infty \frac{Q(K)}{K^2}dK + \frac{1}{K_0^2}\int_{K_0}^F (K-F)dK \\
= e^{rT}\int_0^\infty \frac{Q(K)}{K^2}dK - \frac{1}{K_0^2}\frac{(K_0-F)^2}{2}
\end{aligned}$$

Where $Q(K)$ and $K_0$ are as above in (0) and $F = K_0 + e^{rT}\big(C(K_0) - P(K_0)\big)$. Discretization leads to the formula for valuing the VIX Index.

The above Formula represents a static replication of the variance; the discretization is an approximation of a static hedge.

## 1.4 VIX Futures

In March 2004, the COBE introduced Futures on the VIX Index. Originally with monthly expirations, the CBOE added weekly Futures in 2015. Expirations are usually on a Wednesday morning, 30 days before a Friday settlement for SPX options. VIX Futures are cash settled.

Up until 2010, the average daily volume ("ADV") was rather modest, but has since picked up substantially. While the ADV in 2009 was at 5,000 contracts, it rose to 17,000 in 2010 and reached 205,000 in 2015 and 295,000 in 2018. In particular, the launch of additional products like options, the introduction of various volatility-strategy-indices (see section 1.6) and the launch of ETPs (often tracking the volatility-strategy-indices, see section 1.7) contributed a great deal to the rise in trading activity for VIX Futures.

**The Pricing of VIX Futures**

In terms of pricing, as the VIX index constitutes the square root of a 30 day variance swap, which is priced off exchange traded SPX options, VIX Futures are contracts for a forward payout on the VIX index and hence a forward on the square root of variance (which is different from the square root of forward-variance). Consequently, the convexity adjustment (the square root of variance is a concave function of variance and hence requires a negative convexity adjustment) lifts the value above that of a pure volatility forward, which is still lower than the corresponding forward variance swap. The negative convexity adjustment from the forward variance swap will increase with volatility of variance and hence the Futures will drop relative to forward variance swaps if volatility of variance increases – i.e. VIX Futures are short volatility of volatility. VIX Futures can be estimated (theoretically) by SPX options calculating the implied forward variance from the expiration of the VIX Futures to 30 days out, i.e. bracketing 30days after the futures expiration date with SPX options to calculate the T+30days variance and subtract the T time variance which can be estimated by bracketing T with SPX options. More specifically:

$$VIX\ Fut\ (T) = 100 * \left[\left({}^{365}\!/_{30}\right) * (\sigma^2_{T,T+30} - conv(T))\right]^{1/2}$$

With $\sigma^2_{T,T+30}$ being the forward, T into T+30days variance, obtained from SPX option data and conv(T) is a convexity adjustment due to taking the square root. The convexity adjustment can be derived from the data on the variance swap futures as traded on the CBOE.

There is no arbitrage relationship between VIX Index and VIX Futures; they are only connected by a nonlinear, statistical relationship (convexity adjustment). While fairly accurate pricing of VIX Futures can be derived from market data and empirical distribution of the time series of Futures prices, deviations from that value may be long lasting and are difficult to arbitrage.

**The delta of VIX Futures**

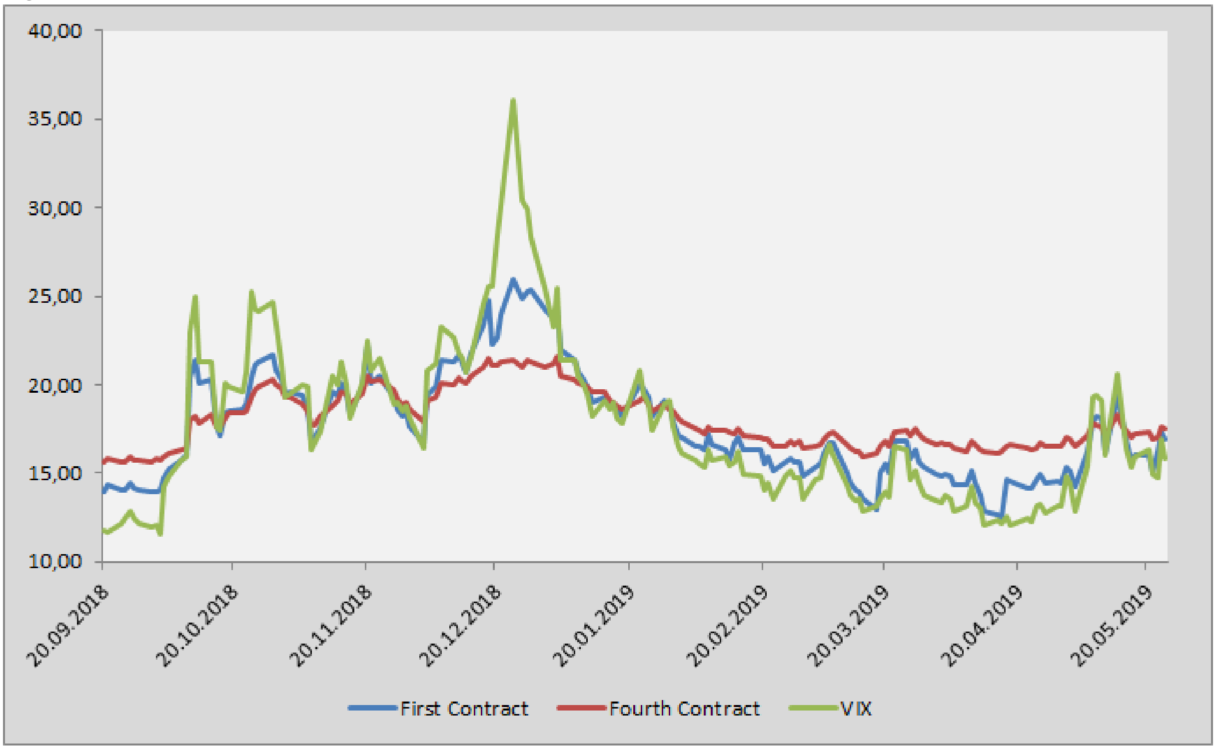
Due to the mean reversion nature of the VIX index and the lack of an arbitrage relationship, the Futures will react with a dampening effect on VIX index moves. The further out a contract, the higher the dampening and the lower the participation in the move of the VIX index. The participation rate of the different Futures w.r. to VIX Index changes is commonly referred to as the Delta of the VIX Futures. Over long periods (and mostly calm market phases), they typically drop from an 70-80% Delta for the front month to a 50-60% delta for the 3rd/4th contract and even lower for contracts further out. This effect hinders volatility trading with Futures: The desire for high delta to closely express an opinion on the VIX index favors short Futures expiries. This narrows the time frame within which the specific opinion has to play out.

In addition, the delta for a specific contract varies with time and state of the market: If only time goes by, the Delta will increase as the futures converge towards the VIX index. However, if the VIX is at very high levels (at the troughs of major SPX downturns), even the front month delta will drop to very low values, anticipating in advance an upcoming drop in volatility and expressing the low probability of further volatility increases. This effect takes place sooner for the long contracts (e.g. 6 month), as the likelihood for a long duration of extreme volatility is low.

Below are the Deltas for the first and fourth contract from September 2018 to June 2019, a period of high volatility and a severe downturn for the SPX, followed by a strong up-move for the SPX and a swiftly falling VIX Index: The Deltas for the full period are 52% and 18% for the first and fourth contract respectively. In the high volatility period from October 2018 to January 2019, the first Delta drops to 47%, while the fourth drops to 16%, for the low volatility regime from January 2019 to June 2019, the Deltas rise to 60% and 23% respectively. The effect of lower Deltas throughout volatile market phases and higher Deltas in up-phases of market/index moves is typical and is further illustrated by the following graph (Fig. 2.1), which illustrates the path of the VIX Index, the first VIX Futures contract and the fourth VIX Futures contract for this time period.

| | First Contract | Fourth Contract | VIX |
|---|---|---|---|
| | Oct 18- Jun 19 | Oct 18- Jun 19 | Oct 18- Jun 19 |
| StDev: | 15,93 | 6,18 | 26,39 |
| Correl: | 86% | 79% | 100% |
| Beta: | 52% | 18% | 100% |
| | Oct 18- Jan 19 | Oct 18- Jan 19 | Oct 18- Jan 19 |
| StDev: | 19,35 | 7,39 | 34,98 |
| Correl: | 85% | 76% | 100% |
| Beta: | 47% | 16% | 100% |
| | Jan 19 - Jun 19 | Jan 19 - Jun 19 | Jan 19 - Jun 19 |
| StDev: | 13,77 | 5,35 | 20,10 |
| Correl: | 88% | 85% | 100% |
| Beta: | 60% | 23% | 100% |

Fig.: 2.1




While the VIX spiked from 15% to 35% in December of 2018, the Futures showed a relative calm response. In the following period of strong index up-trend and falling volatility, the paths of the Futures follow the path of the VIX much more closely.

**The Shape of the Futures Curve**

The interplay between various Deltas and market consensus views on future expected volatility levels has further effects: In “normal” times of volatility slightly above or below the long-term mean, the contracts will increase with expiration times, i.e. longer contracts will be priced with a premium to shorter contracts. It reflects a higher likelihood of market disruptions for longer horizons. In this case, the curve is said to trade in contango. Only during major market disruptions, when the VIX Index has risen fast and trades above the front months, will the Futures curve start to follow the move up, with decreasing Deltas for increasing expirations and hence build out a declining shape with higher expirations, called backwardation.

Most of the time, the curve is priced in contango, which is the source for widely employed curve trades, that monetize the shape of the curve and the fact that the contracts roll down the curve as time goes by: If the level of volatility is unchanged, the contracts will trade down due to shorter expiration times, slowly converging to the VIX Index at expiration. Strategies to exploit this effect will be discussed further below.

Below is the VIX Futures curve, as traded on June 7, 2019. The graph is taken from www.vixcentral.com. Here, only the first 5 contracts trade in contango, with a following bump for November and December, expressing the view that implied volatility might be lower in the last 2 months of 2019 (and consequently the view of an SPX rise during that time).

Fig.: 2.2

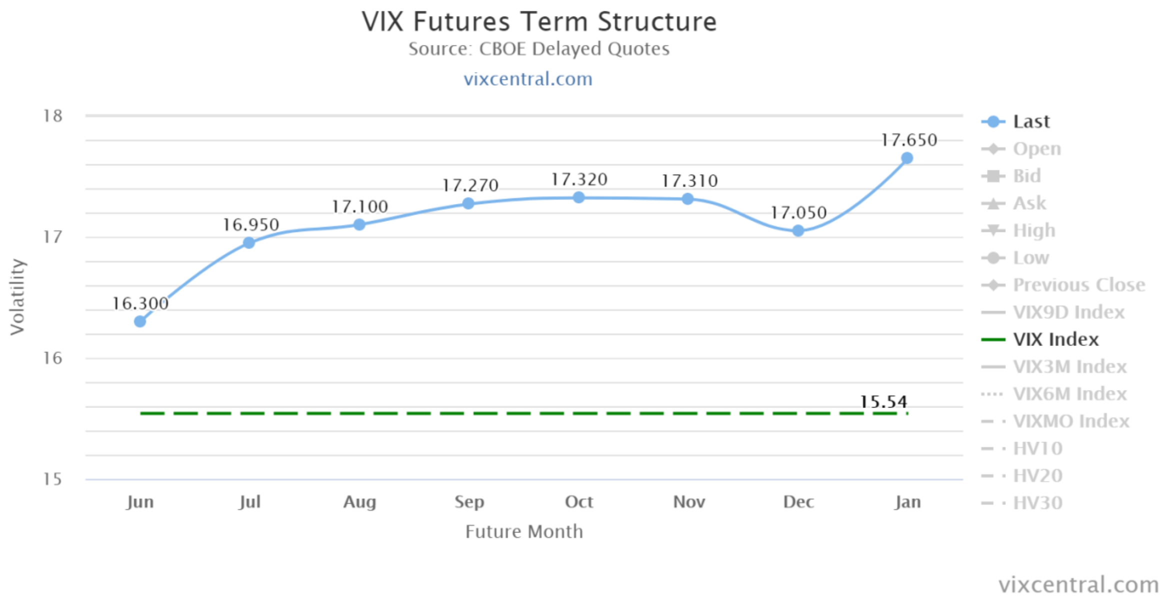
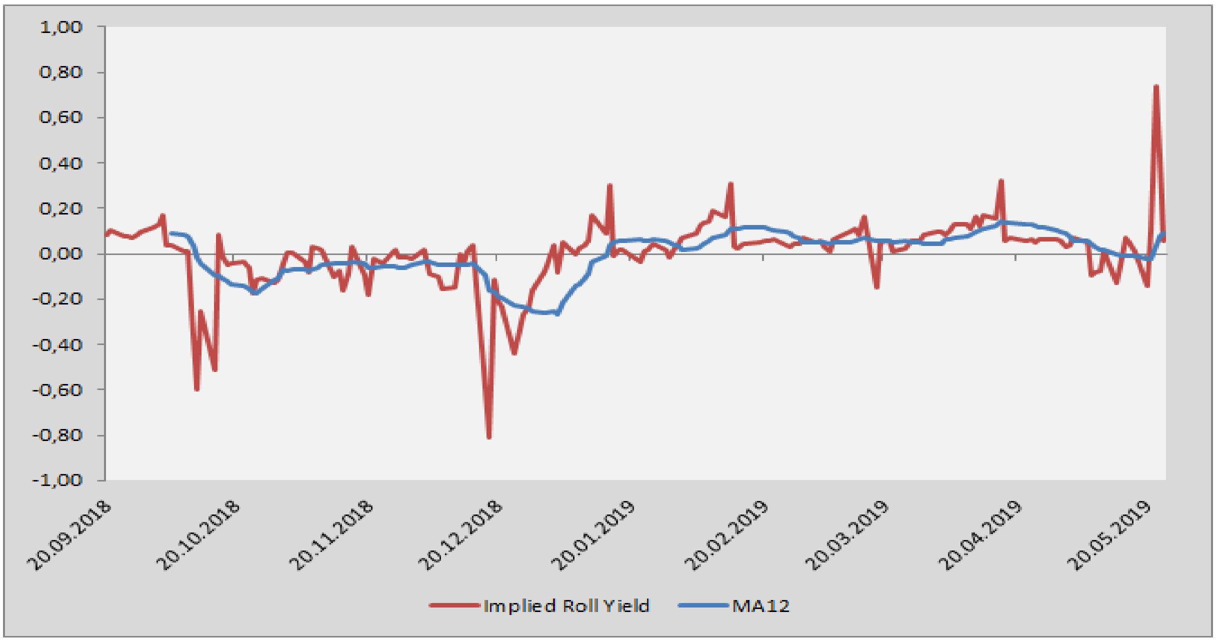



Fig 2.3 below illustrates the Roll yield, the speed of convergence to the VIX Index of the front contract for the period from September 2018 to June 2019: It measure daily the difference between the front contract and the VIX Index averaged over the life of the contract and its 12-day moving average.



**Historical Curve Shapes: Some Examples around major Market Disruptions**

To illustrate how the curve can be affected by turbulent events, the shape of the curve several weeks before and after the culmination of the Lehman crisis is shown in Fig 2.4 While the curve was steep in the front and basically flat thereafter in mid-August, the shape of the curve had changed dramatically by mid-October with backwardation over the full curve trading at extreme levels and spreads for the front end.

Fig.: 2.4:

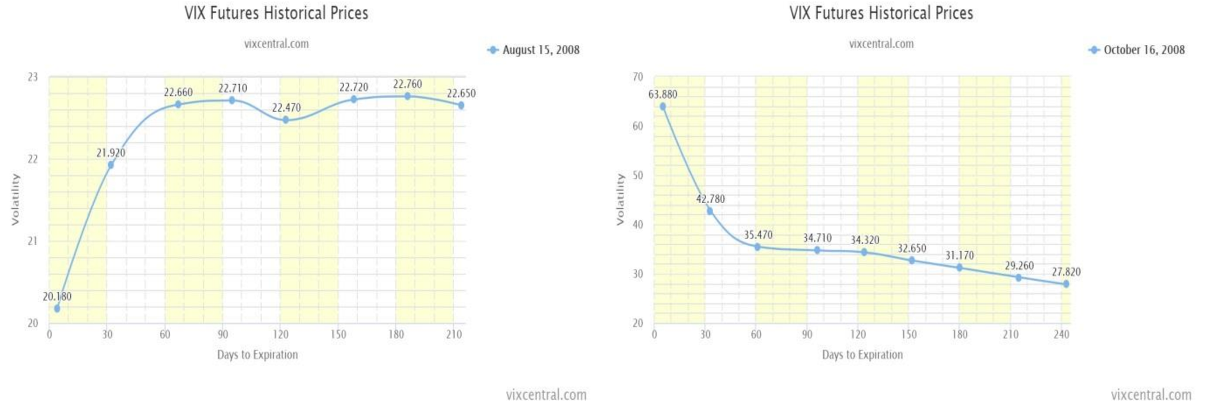
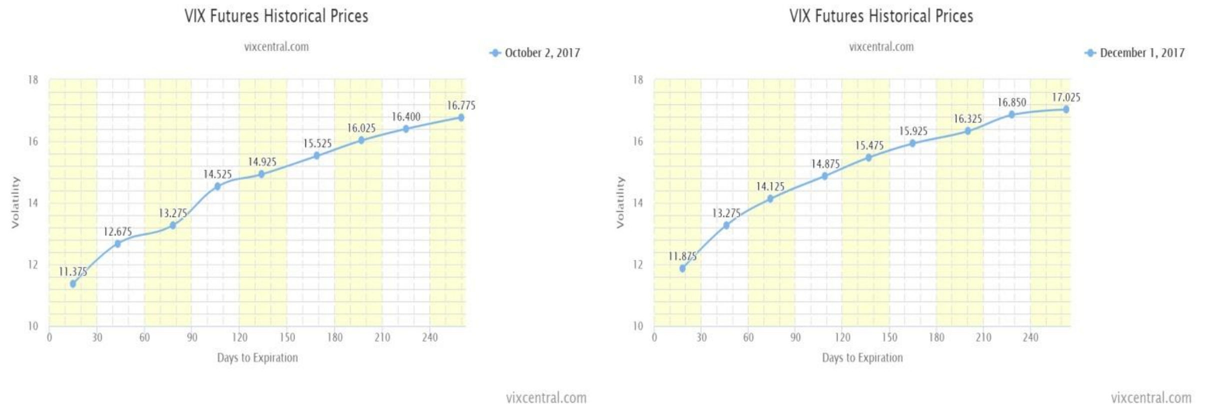


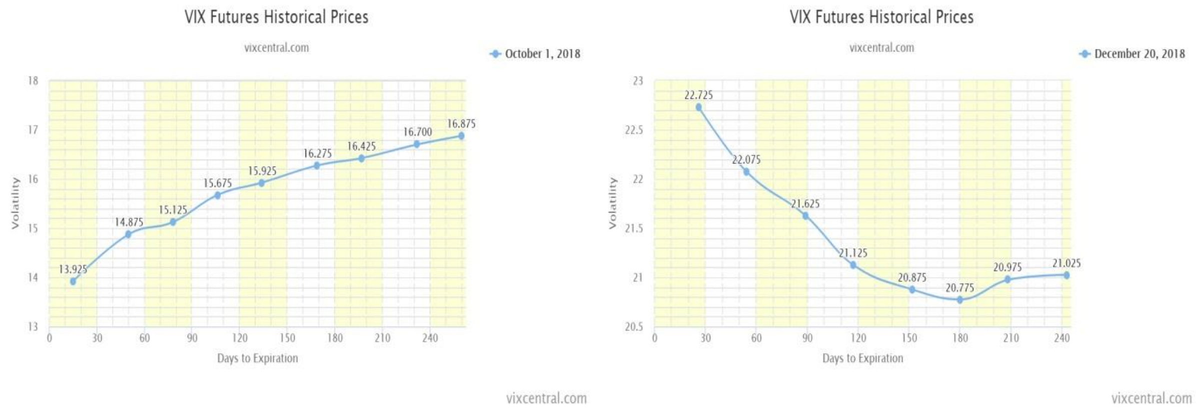

Fig 2.5 reflects the shape of the curves in October and December 2017, as an example of a typical shape in calm and up-wards drifting SPX market-phases. The whole year of 2017 was characterized by low volatilities and almost permanent contango of the VIX curve.

Fig.: 2.5:




The last two graphs illustrate the VIX curves at the beginning of October 2018, the time the US indices reached all time highs, and December 2018, after two months of pretty hefty selling for equity markets. While the curve on October presented a fairly steep contango, the December curve shows a full backwardation although at modest levels given the selloff in the underlying SPX.

Fig.: 2.6:




## 1.5 VIX Options

VIX Options were introduced on February 24, 2006. VIX Options are cash settled contracts on the VIX Index, and are settled against the calculation of the VIX Index through a special opening quotation ("SOQ") process (same as for VIX Futures) of the SPX Options according to (1) (section 1.2) on the expiration date of the VIX Options. As VIX Option expiration dates coincide with the expiration dates of VIX Futures, the settlement can be viewed as against the Future contract of the same expiration, as the expiring Futures contract has converged to the VIX Index by the time of settlement. VIX Futures rather than the VIX Index reflect the path of the Option-underlying; the Put-Call-Parity for VIX Options is valid against VIX Futures not against the VIX Index. Consequently, the Futures should be the base for modelling and analytics as the proper underlying (the Delta should be the derivative w.r. to the VIX Future, which itself has a (volatile) Delta w.r. to the VIX index)

Options on the VIX index are options on implied volatility in contrast to options on variance swaps, which are options on realized volatility. Beyond options strategies on pure implied volatility they offer, in conjunction with options on variance swaps the implementation of option strategies on the volatility premium.

Volatility on implied is typically lower than volatility on realized (peaks and troughs are more pronounced for realized volatility) and hence options on VIX are typically cheaper than options on variance swaps, however, to monetize this effect the difference in underlying forward prices due to the convexity adjustment has to be taken into account. Also, the realized volatility of VIX Futures increases with time (as the contract moves closer to expiration, its Delta is increasing), the implied volatility of VIX increases in line with that.

The pricing of VIX Options should differ from the pricing of other, in particular Index options. As a start, the underlying is forward volatility and has a strong mean reverting feature. The mean reversion becomes stronger with longer maturities. This is not properly reflected in most pricing models. The valuation models used by market participants range from Whaley's approach to use a GBM for the underlying and a Black model for pricing to square root jump diffusion models.

VIX Options also differ in their behavior from other options: Their Delta is with respect to the corresponding Futures contract, which again has an occasionally very volatile Delta against the Index. It may happen that the VIX Index rises sharply, but due to low Deltas in the Futures and strong mean reversion, the Calls on VIX fall in value at the same time. Due to a volatile Delta in the corresponding Futures, Theta and Gamma of the options tend to be very volatile as well.

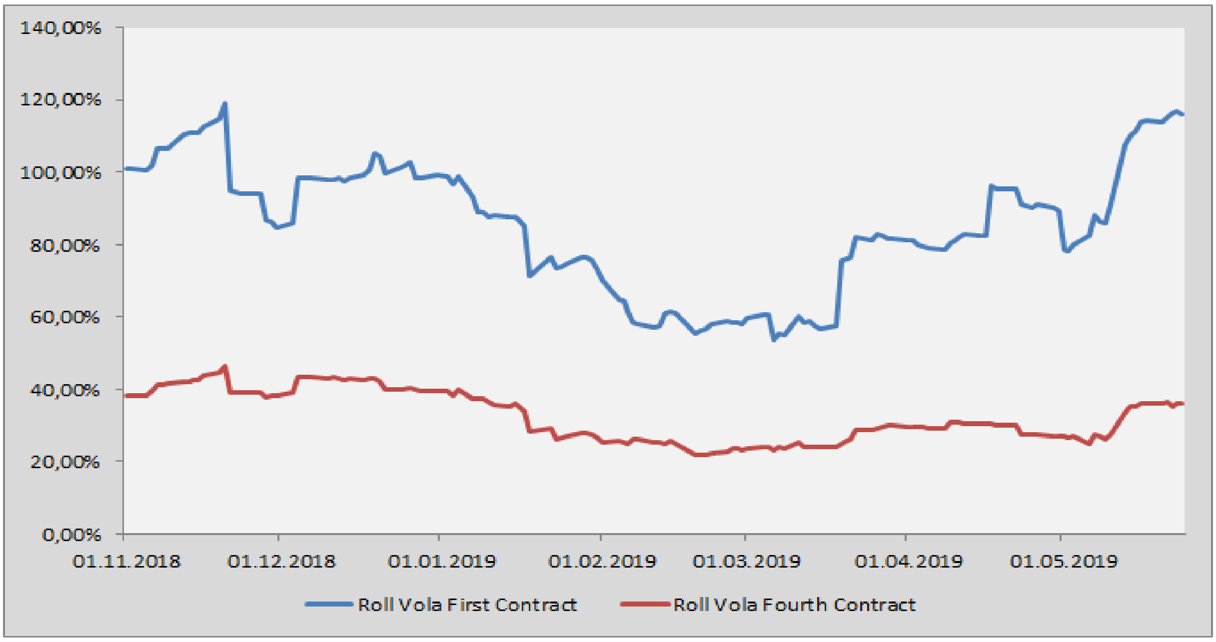
Effects along the Futures curve may admit calendar spreads on VIX Options with a credit, the evolution of these spreads can be very volatile as changes in the curve affecting the performance of the spread can occur very fast.

Some similarities to SPX options however do exist: The signature of a Put Skew for SPX Options is reflected in a more pronounced skew for VIX Calls

The following table reflects the option-chain for VIX Options with expiration on July 5, 2019. The implied forward price can be recovered from this data according to (0), section 1.2.

| Calls | Last | Chg | Bid | Ask | Vol | Open Int | Root | Strike | Puts | Last | Chg | Bid | Ask | Vol | Open Int |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jul 05, 2019 | 2.85 | 0.59 | | | 51 | 578 | VXX | 24 | Jul 05, 2019 | 0.17 | -0.26 | | | 9093 | 10363 |
| Jul 05, 2019 | 2.12 | 0.11 | | | 73 | 334 | VXX | 24.5 | Jul 05, 2019 | 0.28 | -0.27 | | | 399 | 25393 |
| Jul 05, 2019 | 2.08 | 0.34 | | | 607 | 570 | VXX | 25 | Jul 05, 2019 | 0.40 | -0.34 | | | 1195 | 5789 |
| Jul 05, 2019 | 1.79 | 0.43 | | | 330 | 3403 | VXX | 25.5 | Jul 05, 2019 | 0.65 | -0.33 | | | 757 | 867 |
| Jul 05, 2019 | 1.53 | 0.22 | | | 1248 | 922 | VXX | 26 | Jul 05, 2019 | 0.90 | -0.37 | | | 494 | 3747 |
| Jul 05, 2019 | 1.19 | 0.22 | | | 843 | 728 | VXX | 26.5 | Jul 05, 2019 | 1.10 | -0.51 | | | 345 | 3125 |

The following graph reflects the (unsmoothed) rolling realized 30 day volatility of the rolling first- and fourth Futures contracts from November 2018 to June 2019. The volatility of the first contract is higher and reacts stronger to market changes. The implied volatility for the first- and fourth Option expirations follows these paths.



## 1.6 VIX Strategy Indices

The CBOE has developed a number of Strategy Benchmark Indices reflecting the performance of passive, rolling strategies in Futures and Options. Two of them are the VARB-X (VTY), which is based on selling realized volatility via 3-month variance swaps on the S&P 500 and the VIX Premium Strategy Index (VPN), which is based in selling 1-month implied volatility on the S&P 500 via selling the front month of the VIX Futures successively (see more in section 2 ).

S&P Dow Jones ("SPDJ") is calculating and maintaining the S&P VIX Futures Index Series, a collection of VIX Futures based strategy indices, which can be used as benchmarks for proprietary strategies and/or tracking strategies. A number of VIX based ETPs track a VIX Strategy index. This section will only briefly describe some of the strategy indices, for more information see e.g. S&P Dow Jones, (2018).

- **S&P 500 VIX Short-Term Futures Index**: The index seeks to mimic a VIX Futures position with constant 30 days to expiration. It holds positions in the first and second VIX Futures, which it dynamically adjusts to achieve a weighted average expiration of 30 days. Specifically, it holds positions in the first contract, which are continuously rolled into the second contract. The index is tracked by several VIX ETNs (e.g. the "VXX").
- **S&P 500 VIX Mid-Term Futures Index**: The index seeks to mimic a VIX Futures position with constant 60 days to expiration. It holds positions in the fourth, fifth, sixth and seventh VIX Futures, which it dynamically adjusts to achieve a weighted average expiration of 60 days. Specifically, it holds a third of the total position in each of the fifth and sixth contract and continuously rolls the last third of the total position from the fourth to the seventh contract. The index is tracked by the "VXZ" ETN.
- **S&P 500 Dynamic VIX Futures Index:** The index ("DYN") allocates between the VIX Short-Term Futures Index ("ST") and the Mid-Term Futures Index ("MT"), with dynamically adjusted portfolio weights, depending on the shape if the VIX-Futures curve.

  $$DYN = \omega_S ST + \omega_M MT$$

  The portfolio weights $\omega_S$ and $\omega_M$ are chosen depending on the ratio of the VIX to the VXV Index (The VXV index is CBOE's 3 month volatility index, calculated like the VIX index with SPX options bracketing 90days), according to the following table.

  | | VXX | VXZ |
  |---|---|---|
  | IVTS < 0,90 | -0,30 | 0,70 |
  | 0,90 < IVTS < 1 | -0,20 | 0,80 |
  | 1 < IVTS < 1,05 | 0,00 | 1,00 |
  | 1,05 < IVTS < 1,15 | 0,25 | 0,75 |
  | 1,15 < IVTS | 0,50 | 0,50 |

  The Index seeks to capture the roll yield of the ST position by being short while in contango and going long while the curve is in backwardation. The MT position balances the overall Delta. See more on the behavior and the benefit of this Index in the section 2 on "Strategies".
- **S&P 500 VIX Futures Enhanced Roll Index:** The index holds positions in the first five expirations of the VIX Futures. The portfolio weights in the individual Futures are determined by the levels of the VIX index and its 15 day moving average. If the VIX is above its moving average, it is assumed to be in "rising" mode and the curve is assumed to be in backwardation. The weights are then more front loaded and vice versa to profit from the directional moves and to harvest the roll yield.

- **S&P 500 VIX Futures Long/Short Strategy Index ("LS"):** The index builds a portfolio of 13 sub-portfolios, each a combination of a leveraged Short-Term Index ("LEV") and a Short-Term Inverse Index ("INV")
$$LS = \omega_{lev}LEV + \omega_{inv}INV$$
LEV and INV are both strategy indices, which will be explained in the context of ETPs in section 1.7. The index seeks to maintain a target Delta but counters negative roll-yield effects (stay long, specifically to capture or hedge against up-side spikes but capture contango-roll-yield by adding the inverse). For additional comments on the benefits of this strategy, see section 2 on Strategies.

  The S&P VIX Futures Index Series lists a number of other Strategy indices, which will be discussed in general terms in the section 2. The members of the Index Series are quoted by a number of market-data-providers (Bloomberg, Reuters etc.) and can thus be tracked over longer periods to monitor their real-life performance against the stated goal. They can be tracked by any investor, as the recipe for its construction is fully disclosed. Moreover, they can serve as a basis in which investors can base their proprietary strategies by adapting existing indices.

## 1.7 VIX ETPs

The VIX index is not a tradable index, VIX Futures are expiring contracts with a volatile Delta and options on VIX are not suitable for everybody as they are subject to a variety of nontrivial option-specific effects. To facilitate easier positioning in Volatility and to create a non-expiring instrument, Barclays introduced two VIX based ETNs in January and February of 2009. The i-Path S&P 500 VIX short term Futures (VXX), tracking the S&P 500 VIX Short-Term Futures Index (see Section 1.6) and the i-Path S&P 500 VIX mid-term Futures (VXZ), tracking the S&P 500 VIX Mid-Term Futures Index. They both track SPDJ Indices and while the VIX and its Futures reflect the 30 day implied Volatility, respectively the forward 30 day implied Volatility, the VXX and VXZ do not reflect a Volatility level, they simply reflect the current value of the strategy they track. Subsequently issued VIX related ETPs included Barclay's first inverse VIX ETN, the SVXY, issued in 2010, replicating a short Short-Term Futures position and allowing the investor to take short positions in SPX option implied volatility. More inverse products like Credit Suisse' XIV followed as well as leveraged VIX based ETP's. VIX based ETPs are today a well-established asset class, creating and supporting substantial liquidity in the VIX universe. These instruments are not without risk though. The "Volatility-Tsunami" on February 5, 2018, with the closure of the XIV exposed some of the misconceptions, both, on the issuer side in the construction of the ETP as well as on the investor side for not fully understanding the mechanics of the products. Even today, the standard VIX Short Term Futures ETPs (e.g. VXX) are still misunderstood by a lot of market participants.

**VXX and VXZ**

In tracking the Dow-Jones S&P 500 VIX Short-Term Futures Index, the VXX mimics a Futures contract with a constant time to expiration of 30 days. The hedge for the VXX dynamically allocates between the two front Futures, to achieve an average expiration time of 30 days. On each trading day, a fraction in the first contract is closed and a corresponding position in the second Futures is added. The allocation between the two contracts thus shift continuously from the first to the second contract: If the first Futures contract would have initially 30 days until expiration, the VXX hedge would have an allocation of 100% in the first contract. If the number of days until expiration would be 15 days for the first contract and 45 days for the second contract, the allocation would be 50% in each contract, if the first contract expires and the second contract has 30 days in the second contract, the allocation would be 100% in the second contract, which then becomes the new first contract.

The VXX is not perfectly tracking the VIX index. Given that VIX Futures have varying Deltas with respect to the VIX index, with the Deltas declining with increasing expiration, the VXX will also have a Delta of less than 1, i.e. VXX is a dampened version of a VIX index tracker: As the VXX on any given day is a portfolio with a position in the first contract and another position in the second contract, the reaction of the VXX to movements in the VIX index is a combination of the reaction of the first two contracts. Specifically,

$$VXX_t = \alpha_t V1_t + (1 - \alpha_t)V2_t$$

With $VXX_t, V1_t, V2_t$ the VXX, V1 (fírst contract) and V2 (second contract) respectively on day t, $\alpha_t$ the proportion of the VXX position in V1.

Roughly speaking, the position in V1 is reduced daily by 1/30th :

$$\alpha_{t+1} = (1 - \frac{1}{30})\alpha_t$$

The participation in VIX index moves is consequently (as measured by respective Deltas):

$$DeltaVXX_t = \alpha_t \Delta 1_t + (1 - \alpha_t)\Delta 2_t$$

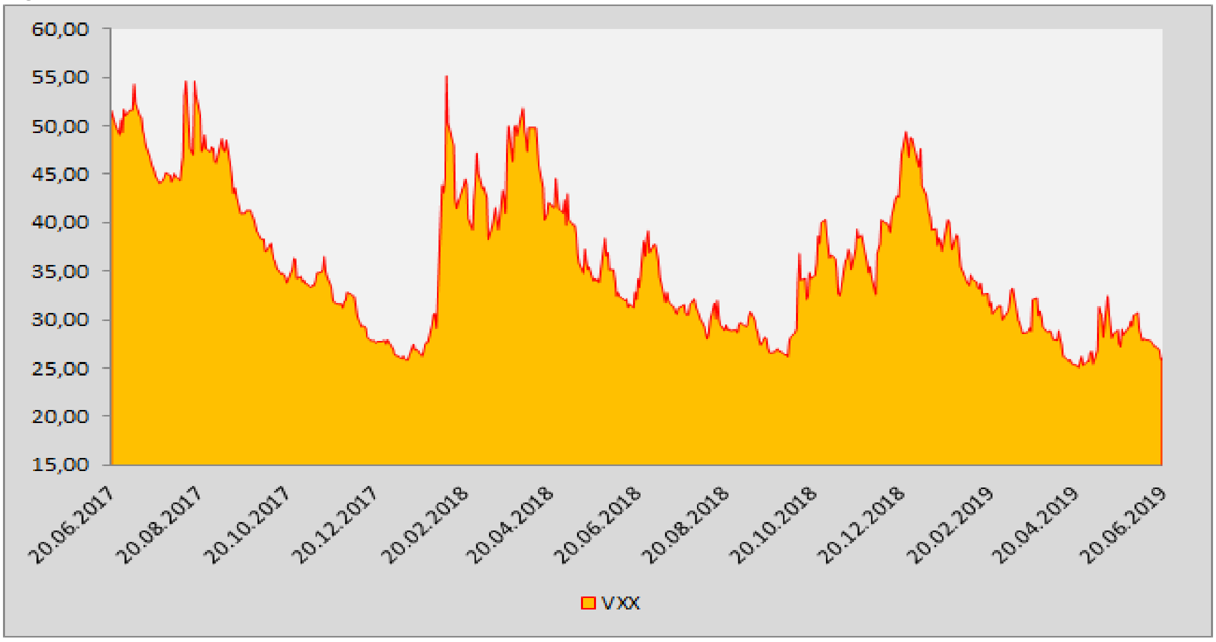
Hence, the VXX typically reacts less strongly to VIX index moves than V1, but stronger than V2, the correlation of the VXX to the VIX index is therefore lower than the V1 correlation to the VIX index, but higher than the V2 correlation.

The VXX is not a true synthetic 30 day Future: The VXX is a linear interpolation between V1 and V2. If the curve is in contango, the curve is concave and thus the proper 30 day Future would be above the linear interpolation.

**The performance of VXX**

The performance of the VXX since its inception is dismal. Specifically, it underperforms the VIX Index substantially. Since its inception, the VXX has lost 95% of its value. Nevertheless, while certainly not suited as a buy and hold investment, it is a very effective hedging and trading tool and as such attracts still very high volume despite its long run performance.

Fig.: 5.1




The reason for the strong underperformance to the VIX index results from the shape of the curve: Most of the time, the curve is in contango. This means, that on top of directional moves, each Futures contract will roll down the curve if time goes by. That is, all else being equal and only time passing by, each Futures contract will lose value, while the curve is in contango (gain value if the curve is in backwardation). The VXX is a portfolio of two positions, both positions lose value due to contango and hence also the VXX will lose value during those phases – a reason that makes inverse VIX-ETPs, which have the opposite performance feature, so popular as buy and hold instruments. Often the reason for the loss of the VXX in contango is attributed to the roll, i.e. the sale of the front contract and purchase of the second contract – "sell low and buy high". However, as the reduction of the front month position and the buildup of the second month position are valued at market prices, the roll has no direct effect on the value of the VXX; the reason is the individual loss of the two VXX positions independent of the re-allocation between the two Futures.

The overall daily change in VXX is

$$\begin{aligned} VXX_{t+1} - VXX_t &= DeltaVXX_t(VIX_{t+1} - VIX_t) + \alpha_t RollYieldV1 + (1-\alpha_t)RollYieldV2 \\ &= [\alpha_t \Delta 1_t + (1-\alpha_t)\Delta 2_t](VIX_{t+1} - VIX_t) + \alpha_t RollYieldV1 \\ &\quad + (1-\alpha_t)RollYieldV2 \end{aligned}$$

The mechanics for the VXZ works the same way, except that 1/3$^{rd}$ of the position is in each of the 5$^{th}$ and 6$^{th}$ contract respectively, while the remaining 1/3$^{rd}$ is split between the 4$^{th}$ and 7$^{th}$ contract in the same way as with the 1$^{st}$ and 2$^{nd}$ contracts for the VXZ. The Delta and correlation can be derived accordingly and are, a reflection of the Deltas and correlation of the underlying contracts lower than for the VXX, the performance of the VXZ is not a as negative as for the VXX.

Given that contango is the shape of the curve in over 80% of the time (even 2008 the VIX curve was in contango over 50% of the calendar year), the VXX performance in the long run is dominated by a steady downdraft caused by the contango. Only during turbulent market phases for the SPX, does the VIX curve change into a downward shaped, backwardation curve.

This negative carry feature of VXX in normal market phases is similar in effect, but different in nature, to time decay in long options positions. One of the reasons why the VXX is still popular is similar to the popularity of buying options during market turbulences: The VXX reacts with sharp spikes to major down-moves in SPX

However, the VXX hedge acts against its own interest: Given the size of the VXX or similar ETPs, the roll action of the hedge exerts a constant downward pressure to the front and upward pressure on the second contract, widening the spread and supporting contango.

**Inverse and Leveraged Exchange Traded Products on VIX**

The hedging strategy of invers and levered products on VIX is the same as for other underlying. There is a distinct difference in the maintenance compared to unlevered products like the VXX or VXZ: While unlevered VIX ETPs need only constantly shift the hedge between different contracts to maintain a constant average life, levered and inverse products need to take additional hedge adjustments to keep a constant leverage ratio.

Leveraged ETPs (ETNs/ETFs) with a leverage factor of L (including L=-1 for inverse products) provide the investor typically with a daily return of L times the daily return of the underlying. This way of leveraging differs from a standard L-times levered notional amount.

Table 5.1 illustrates, what happens to different forms of leveraging an index return for a set of three different 6-day return scenarios.

The first scenario assumes a return of 10% each day for the underlying index. A simple investment in the index will start at 100 and end the 6 day-period with a value of 177 for a total return of 77%. This is illustrated in the first two columns of the first part of the table.

A standard 2 times leverage of notional investment will invest 100 but reflect the return by a position of 200 of the index. This position will be hedged (or created) via Futures or Index Swaps with an exposure of 2 times the underlying index. For the first day, the value of the hedge will jump from 200 to 220 for a gain of 20 and hence the value of the levered investment will jump from 100 to 120, a return of 20%. On the second day, the value of the hedge will jump from 220 to 242 for a gain of 22; the value of the levered position will rise from 120 to 142, a return of 18.33%. The total 6-day return is 154%, twice the index return for that period. This is illustrated in columns 3, 4 & 5.

A levered investment with a targeted return of 2 times daily index return will be initially hedged as before, via Futures or index swaps on twice the underlying index. The first day return will also be as before and return the correct 2 times index return for the day. The value of the levered investment will be 120. However, just leaving the hedge unchanged at twice the original underlying will only produce a return of 18.33% for the second day. In order to achieve a return of 2 times daily return on the second day, the hedge notional needs to increase to 2 times the value of the levered investment at the end of the first day, not simply 2 times the value of the underlying asset, i.e. to 2 x 120 = 240 instead of 2 x 110 = 220. That means the hedge notional needs to increase from to 220 to 240, i.e. by 20. The same logic applies to the following days: Each day, the levered notional needs to further increase in order to maintain a constant leverage of 2 times daily return. The total period return amounts to 199% and exceeds the return of a simple 2 times (notional) levered investment. This is illustrated in columns 6, 7 & 8.

The same logic applies to inverse products: Columns 9, 10 & 11 illustrate the daily performance development for a short investment in the index, columns 12, 13 & 14, the performance path of an investment returning -1 times the daily index return. Here, as before, the notional of the hedge needs to be adjusted each day to maintain a constant daily leverage of -1. In essence, a standard levered product will always keep the same leverage with respect to the value of the underlying; the daily levered product will keep the leverage with respect to itself, i.e. the notional of the levered product instead of the underlying.

Below the first scenario of 6 daily returns of 10%, the following tables illustrate different scenarios for all the different investments.

Table 5.1

| Index Notional | Index Return | Twice Index Hedge Not. | Twice Index Value | Twice Index Return | Twice Daily Hedge Not. | Twice Daily Value | Twice Daily Return | Short Hedge Not. | Short Value | Short Return | Daily Inv Hedge Not. | Daily Inv Value | Daily Inv Return |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100% | 0 | 200% | 100% | 0 | 200% | 100% | 0 | -100% | 100% | 0 | -100% | 100% | 0 |
| 110% | 10% | 220% | 120% | 10% | 240% | 120% | 20% | -110% | 90% | 9% | -90% | 90% | -10% |
| 121% | 10% | 242% | 142% | 10% | 288% | 144% | 20% | -121% | 79% | 9% | -81% | 81% | -10% |
| 133% | 10% | 266% | 166% | 10% | 346% | 173% | 20% | -133% | 67% | 9% | -73% | 73% | -10% |
| 146% | 10% | 293% | 193% | 10% | 415% | 207% | 20% | -146% | 54% | 9% | -66% | 66% | -10% |
| 161% | 10% | 322% | 222% | 10% | 498% | 249% | 20% | -161% | 39% | 9% | -59% | 59% | -10% |
| 177% | 10% | 354% | 254% | 10% | 597% | 299% | 20% | -177% | 23% | 9% | -53% | 53% | -10% |
| Total: | 77% | | Total: | 154% | | Total: | 199% | | Total: | -77% | | Total: | -47% |

| Index Notional | Index Return | Twice Index Hedge Not. | Twice Index Value | Twice Index Return | Twice Daily Hedge Not. | Twice Daily Value | Twice Daily Return | Short Hedge Not. | Short Value | Short Return | Daily Inv Hedge Not. | Daily Inv Value | Daily Inv Return |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100% | 0 | 200% | 100% | 0 | 200% | 100% | 0 | -100% | 100% | 0 | -100% | 100% | 0 |
| 90% | -10% | 180% | 80% | -10% | 160% | 80% | -20% | -90% | 110% | -11% | -110% | 110% | 10% |
| 81% | -10% | 162% | 62% | -10% | 128% | 64% | -20% | -81% | 119% | -11% | -121% | 121% | 10% |
| 73% | -10% | 146% | 46% | -10% | 102% | 51% | -20% | -73% | 127% | -11% | -133% | 133% | 10% |
| 66% | -10% | 131% | 31% | -10% | 82% | 41% | -20% | -66% | 134% | -11% | -146% | 146% | 10% |
| 59% | -10% | 118% | 18% | -10% | 66% | 33% | -20% | -59% | 141% | -11% | -161% | 161% | 10% |
| 53% | -10% | 106% | 6% | -10% | 52% | 26% | -20% | -53% | 147% | -11% | -177% | 177% | 10% |
| Total: | -47% | | Total: | -94% | | Total: | -74% | | Total: | 47% | | Total: | 77% |

| Index Notional | Index Return | Twice Index Hedge Not. | Twice Index Value | Twice Index Return | Twice Daily Hedge Not. | Twice Daily Value | Twice Daily Return | Short Hedge Not. | Short Value | Short Return | Daily Inv Hedge Not. | Daily Inv Value | Daily Inv Return |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100% | 0 | 200% | 100% | 0 | 200% | 100% | 0 | -100% | 100% | 0 | -100% | 100% | 0 |
| 110% | 10% | 220% | 120% | 10% | 240% | 120% | 20% | -110% | 90% | 9% | -90% | 90% | -10% |
| 99% | -10% | 198% | 98% | -10% | 192% | 96% | -20% | -99% | 101% | -11% | -99% | 99% | 10% |
| 109% | 10% | 218% | 118% | 10% | 230% | 115% | 20% | -109% | 91% | 9% | -89% | 89% | -10% |
| 98% | -10% | 196% | 96% | -10% | 184% | 92% | -20% | -98% | 102% | -11% | -98% | 98% | 10% |
| 108% | 10% | 216% | 116% | 10% | 221% | 111% | 20% | -108% | 92% | 9% | -88% | 88% | -10% |
| 97% | -10% | 194% | 94% | -10% | 177% | 88% | -20% | -97% | 103% | -11% | -97% | 97% | 10% |
| Total: | -3% | | Total: | -6% | | Total: | -12% | | Total: | 3% | | Total: | -3% |

**Daily Rebalancing of Hedges to maintain constant Leverage, a formal Look**

In order to provide a constant L times daily leverage, the hedge position will have to be rebalanced on a daily basis. Specifically for leveraged VIX ETPs, this rebalancing is independent of- and in addition to the daily rolls to provide an underlying reflecting constant average expiration.

In order to provide a constant L times daily leverage, the hedge position initially taken will be a position in L times the underlying performing asset (e.g. via Swaps or Futures). However, as the last table shows for the case of L=2, the subsequent performance of this hedge position and of the target will differ. The reason for this is, that the base upon which further performance builds is different:

To provide with a return of L times the underlying return on the notional of the ETP, the position is L times the ETP notional in the underlying. Starting from an underlying notional of 100%, the ETP will also have an initial notional of 100%, the hedge will have a notional of L times 100%. After the first day with an underlying return of $R_0$ , neglecting financing costs arising out of hedges and ETP fees, the hedge notional will be

$$N_H = 100\% * L * (1 + R_0)$$

The value of the ETP will be L times the underlying return

$$N_E = 100\% * (1 + LR_0)$$

In order for the hedge to provide a leverage of L for the next daily return, the necessary hedge notional will need to be:

$$N_H^* = L * N_E = 100\% * L * (1 + LR_0)$$

Which means the hedge notional $N_H$ will have to adjusted by

$$N_H^* - N_H = 100 * L * (1 + R_0)\text{- }100\% * L * (1 + LR_0) = 100\% * L * (L - 1) * \ R_0$$

This means that a leveraged ETP on VIX undergoes a change of underlying exposure depending on the leverage factor L and the daily return of the underlying. Only for L=1, i.e. an unleveraged index tracker like VXX, will there be no daily rebalancing. Specifically, for L=2, the daily rebalancing of notional will be 2 times the underlying return, for L=3, it will be 6 times the underlying return. For an inverse product with L=-1, the rebalancing will be 2 times the underlying return. In order to fully hedge a leveraged ETP on the Dow-Jones S&P 500 VIX Short-Term Futures Index, there are two types of hedge adjustments: The daily roll to keep a constant average expiration of the underlying and the notional adjustments as just described.

Important to note is, that the rebalancing will always be in the direction of the underlying move, it will be pro-cyclical. Specifically this means, that to hedge an inverse VIX ETP in a rising VIX environment, more VIX Futures will have to be bought to maintain the daily inverse character of the ETP. It is this feature, which drove the XIV to extinction during the “Volatility Tsunami” on February 5, 2018.

**Continuous Rebalancing of leveraged ETPs**

While the previous section illustrated the daily rebalancing of leveraged ETPs, which is the most common way for running the ETP hedges, following other, more frequent or irregular time intervals for adjustments to keep a constant leverage ratio work accordingly. In a continuous adjustment, let $P_t$ denote the leveraged ETP with constant leverage factor of L and $X_t$ the underlying. Constant continuous leverage would translate into (again neglecting financing cost and ETP fees)

$$\frac{dP_t}{P_t} = L\frac{dX_t}{X_t} \quad or \quad dP_t = L\frac{dX_t}{X_t}P_t$$

If $X_t$ follows a GBM, $\frac{dX_t}{X_t} = \mu dt + \ \sigma dW_t$, this is solved by

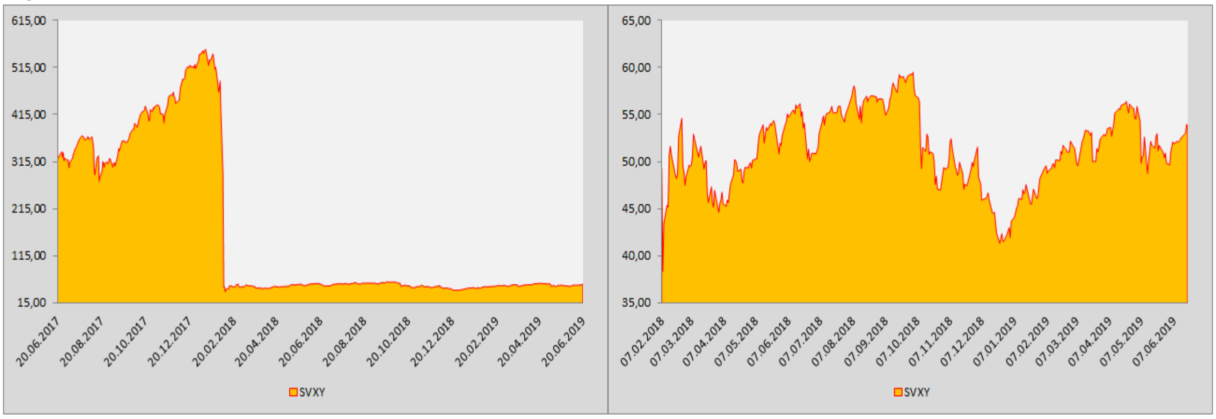
$$P_t = P_0\left(\frac{X_t}{X_0}\right)^L \exp(-\tfrac{1}{2}L(L-1)\sigma^2 t)$$

This can be used to construct intraday positions (intraday adjusted leverage constant against daily adjusted leverage constant), dependent on the intraday realized volatility. Moreover, hedging the leverage factor in irregular spaced intervals can be used to “arbitrage” an existing leveraged ETP against a proprietary hedging strategy. Finally, it can also be used to price options on the leveraged ETP, using the Carr-Madan equation (2), section 1.3. For inclusion of financing costs and fees see, e.g., Zhang (2010).

**The inverse ETPs**

Starting in 2010 several inverse VIX ETPS were issued, essentially all of them tracking the S&P 500 VIX Short-Term Futures Inverse Daily Index, which is not mentioned in Section 1.6, but belongs to the SPDJ Index group on VIX. The most popular and liquid were Barclay's SVXY and CS' XIV, which was terminated in February 2018. The SVXY is - almost - the opposite to the VXX: It's performance is characterized by a steady climb outside of market turbulences for the same reasons, that cause the downdraft in VXX. But in the same way that makes the VXX a very effective hedge against turbulences, the SVXY bears substantial tails risks, which seem to be more pronounced as the tails risk of short-Put positions in the SPX. The SVXY lost 90% in a single day on February 5, 2018, which led the initiator to lower the leverage to -0.5 instead of the original -1. The XIV lost 97% of its value on that single day and was subsequently terminated. Contrary to the VXX, for which the hedge is simply a continuous roll from front to second futures, the inverse VIX ETPs need to adjust the notional of the hedge to keep a constant leverage factor (of -1 in this case). As outlined in the previous section, this produces a pro-cyclical flow. Specifically, if the VIX rises, inverse need to further buy VIX Futures to maintain the hedge. Figure 5.2 outlines the performance for the SVXY from June 2017 to June 2019 and secondly from just after the Volatility Tsunami in February 2018 until June 2019.

Fig.: 5.2




The table below reflects the correlations and Deltas of the VXX and the SVXY relative to the VIX Index from June 2017 to June 2019.

Table 5.2

| *Full Period* | VXX | SVXY | *After Feb18* | VXX | SVXY |
|---|---|---|---|---|---|
| Correl | 88% | -50% | Correl | 88% | -31% |
| Delta | 39% | -25% | Delta | 45% | -30% |

**What happened on February 5, 2018? The Volatility Tsunami:** On February 5, 2018, the VIX spiked intraday by over 100%, an event that was subsequently named the "Volatility Tsunami", where "a bad day for a group of high flying volatility based products turned into a devastating decline" as Vance Harwood described it in his blog.

The adjustment of the hedge-notional amounts is typically done near the end of each trading day. Given the nationals outstanding for inverse VIX ETPs at the time, it was estimated, that a 10% rise in the underlying Short-Term VIX Futures index, the hedge for the SVXY would need to buy an additional of $100 Billion for every $1 Billion in notional (see Harwood (2018, 2019). In addition other inverse as well as (positively) leveraged ETPs had similar hedging requirements. By then, major moves in the VIX and the underlying of ETPs produced hedging requirements, that amounted to a significant percentage of the entire VIX futures market. On Feb 5, 2018, the VIX Index spiked by over 100%, the underlying index (SPVXSP) for the inverse VIX ETPs by almost 100% albeit less than the VIX index itself. The majority of the spike occurred after 14h NY time. At about 16:15h (NYT) the VIX appreciation for the day reached over 120%, the SPVXSP close to 100% right around the time when ETP hedge adjustments were supposed to be undertaken. The hedging resulted in a 97% loss for the day for the XIV, the ETP was terminated the next day. There is evidence, that the SVXY managers stopped the hedging process and postponed until the next day, on which the VIX dropped and helped the survival of the SVXY. The leverage of the SVXY was subsequently changed from -1 to -0.5. With this lower leverage, XIV would have survived the Volatility Tsunami.

### 1.8 Options on VIX ETPs

For the VXX, UVXY (2 times daily return of the VIX short-term Futures index) and the SVXY (as well as various other VIX ETPs), Nasdaq lists options, that are physically settled into the underlying ETP (like stock options). They provide accessible trading tools for Options strategies to the institutional and professional investor. Given the complex behavior of the underlying as a dynamically adjusted position in VIX Futures, they allow for more differentiated strategies than simple stock or index options. This gives rise to more risks but also potential for additional and unconventional rewards. Specifically, options on UVXY, with their dependence on implied forward volatility of SPX, implied as well as realized volatility on this implied forward volatility provides a rich and challenging playground for the serious option trader.

## 2. Strategies

Trading volatility may serve multiple purposes. On one hand, volatility can be traded directionally on its own right. With the multitude of instruments available, more elaborate strategies beyond the simple long/short can be constructed and implemented. Besides pure directional trading, active curve trading and passive, curve-based carry strategies are some of the more popular ways to employ the universe of VIX instruments.  Some of those strategies have already become crowded. Besides trading volatility as a separate asset class with its own set of strategies, VIX based instruments lend themselves as efficient hedging instruments against SPX sourced risk, as a set of valuable tools in managing a SPX options portfolio or as signaling processes for trading in other asset classes.

### 2.1 Directional Trades

Directional trades in volatility can be implemented by going long VIX Futures or ETPs like VXX or VXZ. There is one major difference to most other underlying: What investors typically want to go long or short is the VIX index itself, for which there is no direct instrument. All available instruments have Deltas of typically less than one, some – long term Futures – substantially less than one. Long VIX positions suffer from curve effects mentioned before. The front months typically combine a higher Delta with a more negative carry effect. Consequently, long directional bets on VIX have to balance out the advantage of having high Delta-participation in VIX index moves against the carry effect from the curve in deciding where along the Futures-curve a position is to be established. If the front month for high participation but high negative roll yield is chosen, an impeccable timing is required. Positioning on the short side, typically done after volatility bursts may then also encounter negative carry due to the inverse VIX curve in those phases, but the inversion typically does not last long before the mean reversion of the VIX normalizes the curve and pulls it back down.

For the timing of directional trades, a number of proprietary entry- and exit signals are used. Signals employed by the S&P Dow Jones Strategy Indices include a signal depending on the VIX Index to its 15 day Moving Average and a measure of the curvature of the VIX curve:

$$c_t = sign\left[\frac{V2 - V1}{V2} - \frac{V7 - V4}{3 * V7}\right]$$

The SPDJ-Indices use information on the recent $c_t$-path to switch allocations between VXX and SVXY with weights of $\pm 1$.

Overall, directional bets are not quite as obvious as for the SPX, given the volatile Delta of the available instruments does not allow for a robust exposure to the moves of the VIX Index. They are also more timing sensitive. For that reason, most trading operations in VIX related strategies prefer non-directional curve based trading themes.

Trading VIX Futures is trading implied volatility, while realized volatility is traded through Variance Futures.

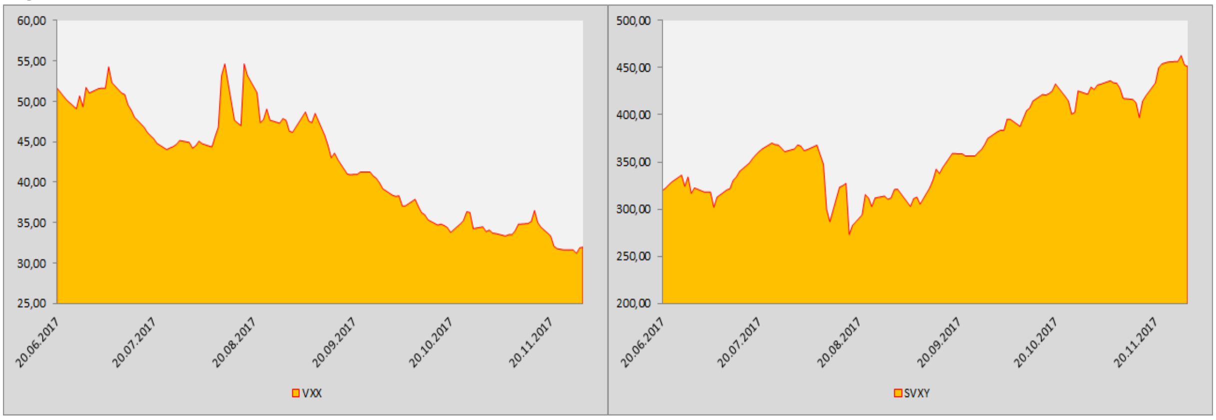
## 2.2 Passive and Semi-Passive Carry Strategies

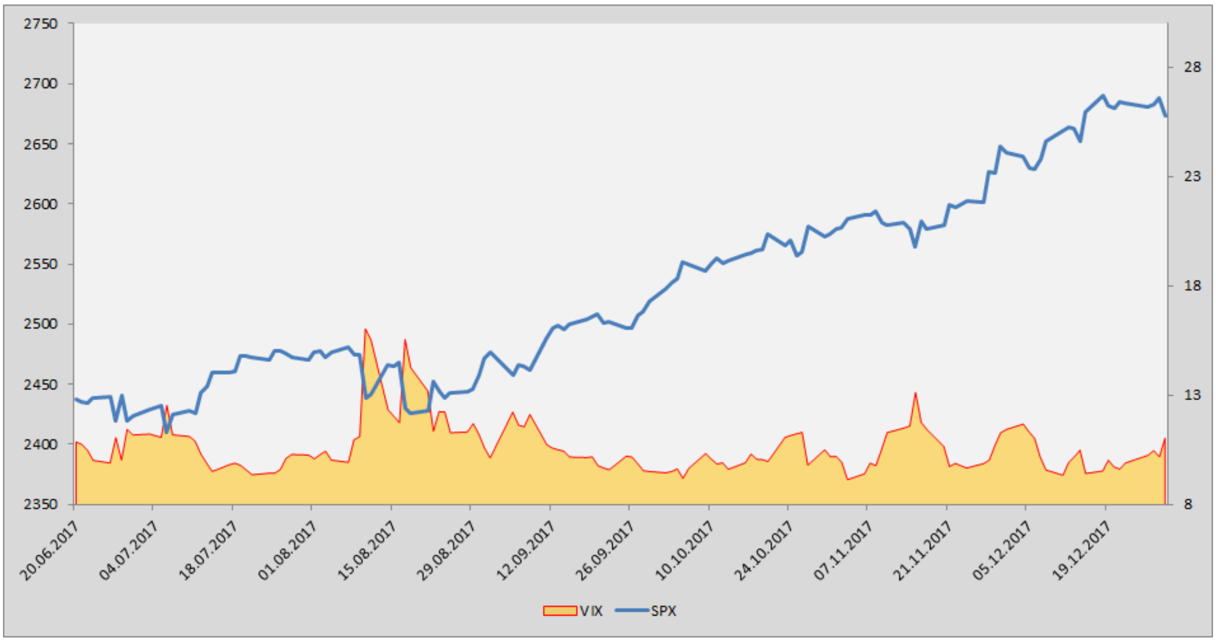
In the absence of major market turbulences, the VIX Index will typically move mainly sideways with a slight downward bias above its long-term mean/equilibrium, with occasional, short-term burst. This is generally accompanied by a normal shape of the VIX Futures curve, i.e. the VIX curve on a typical day is in contango. During short bursts in volatility, the VIX index may for short times rise through the front futures, but often, that motion is reversed before the whole curve has turned into backwardation.

A long position in VIX, as realized by a long position in any of the VIX futures or the VXX ETN will thus always encounter a head wind: They are confronted with the slight downward bias of the VIX and particularly with the steady, negative roll yield, caused by futures converging to the earlier contracts respectively the VIX index. Only when the SPX encounters a major and rapid correction will the curve turn into backwardation.

Figure 2.1 illustrates the performance paths of the SPX, VIX, VXX and SVXY during the calm market phase in H2 2017.

Fig. 2.1:






Specifically the VXX ETN has a time decay or theta characteristic, assimilating that of long, delta-hedged position in Index-Options: Most of the time, there will be a negative drift, turbulent phases however will lead to spikes in the position, which will fade quickly when markets calm again. For delta-hedged index options, this negative drift is caused by the downward bias in implied Volatility in calm periods and by the Volatility premium. For the VXX the causes are the bias in implied Volatility and the shape of the curve also referred to as the curve term-premium. While the bias in implied Volatility affects both positions, a long SPX Option as well as a long VXX position, Volatility premium and curve premium are not identical:

**Volatility Premium vs. Curve Term Premium**

The Volatility premium for SPX options as the difference between implied and ex-post realized volatility is a very robust risk premium, harvested by several option strategies. The two main strategies to monetize the volatility premium are on the one hand, Delta neutral hedged short SPX-Option positions, which leave an unneglectable Gamma risk. This Gamma risk can serve both, as an additional source of reward for a trading inspired approach (mainly on timing of hedging trades), but can also annihilate the Volatility premium. Another approach is provided by a short Variance Swap (or Variance Futures) position, which is solely focused on monetizing the Volatility premium and can be maintained statically, contingent on risk management measures.

The VIX curve term premium (cumulative roll yield) has a similar appearance as the volatility premium in terms of robustness, but is clearly distinct: It arrives mainly from the higher probability of sudden, unforeseen volatility spike in larger time frames during normal times. This higher probability is priced through higher implied volatilities for future volatility, which leads to a rising VIX Futures curve. It reflects only future implied volatility, while the volatility premium reflects the difference of realized to the implied volatility

Volatility premium, directly traded through Variance Swaps, trades Future realized Volatility against spot assessment of current implied Volatility.

VIX curve term premium traded through various term structures strategies, trade future implied volatility at various points in the futures against each other.

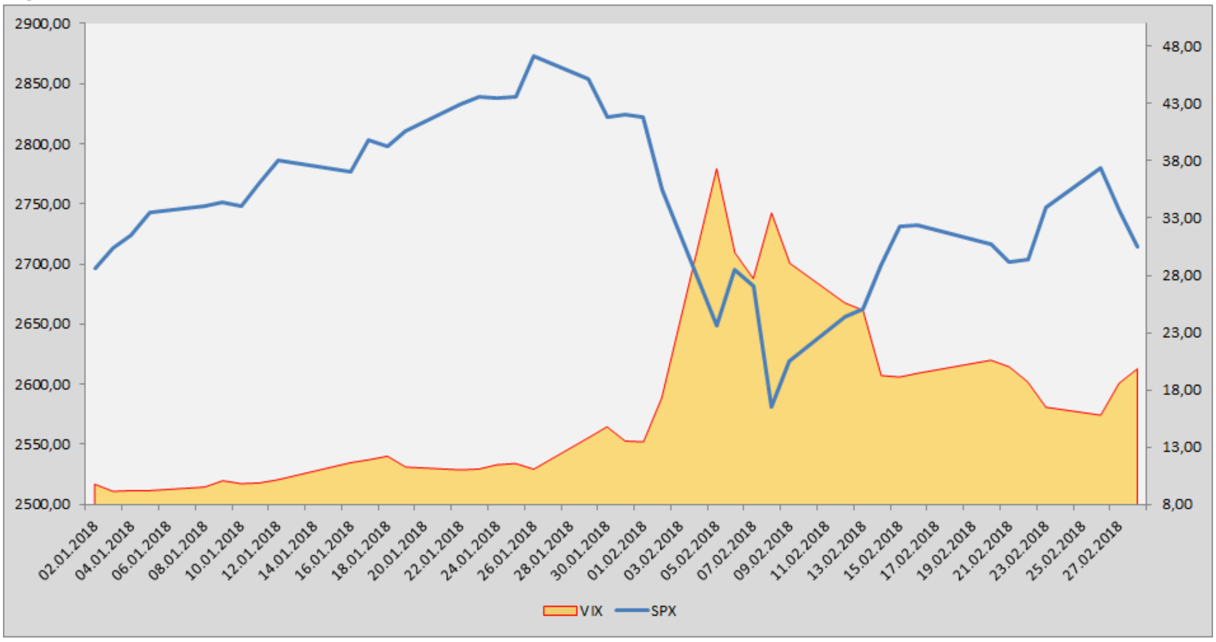
**Carry Strategies with ETPs, the popular Trade**

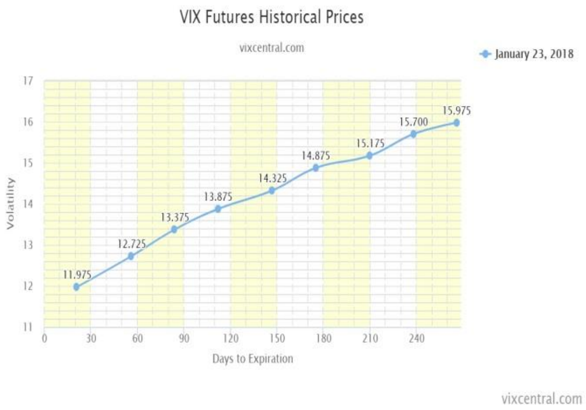
Given the comments earlier on the performance behavior of long VIX related positions, the obvious way to profit from the shape of the curve is to be short the VIX in some form. The easiest way to achieve that is to go long the SVXY, the inverse (factor -0.5) VIX ETF.

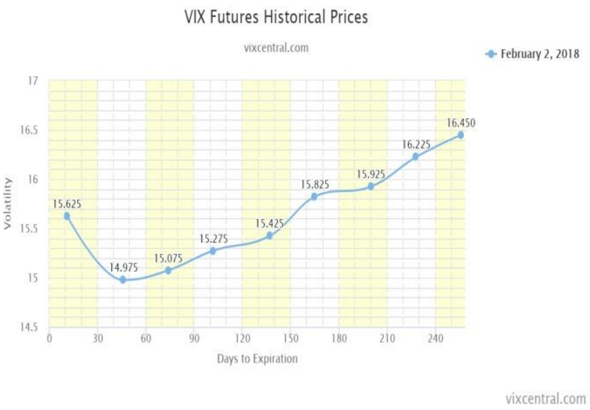
A long position in the SVXY seems to be the most popular if not most crowded trade to capture curve effects in normal and calm markets. The mean reversion tendency of the VIX Index, which pulls the Index down rather fast after up-side spikes, is an additional support for this position. It is not without risk, however, as could be experienced on February 5, 2018. To mitigate the risk of volatility bursts, a stringent stop-strategy or other risk management features to avoid phases of sharply rising volatility needs to be applied. A simple feature would be to exit a long SVXY position as soon as the front part of the curve inverts.

Figure 2.2 exhibits the changes in the VIX curve during the SPX down-move in February of 2018, a period that includes the Volatility Tsunami. The shapes of the curve at various points of time in that period are shown and illustrate the changes from contango (January 23, 2018) to backwardation and back to contango (March 13, 2018). For a long position in SVXY, February of 2018 would have been devastating. Exiting at February 2, with the front part of the curve already in backwardation and entering again in early March, with the curve in contango again would have performed substantially better.

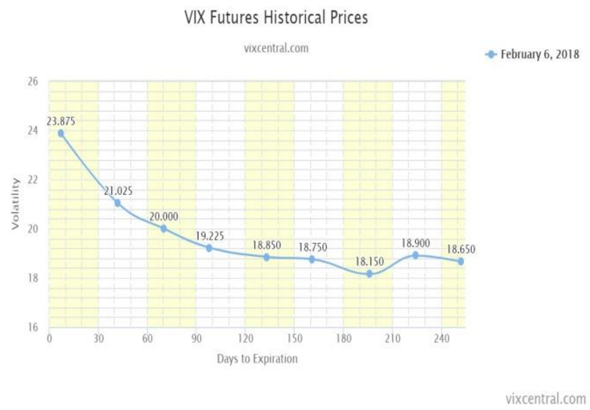
Fig. 2.2:




VIX Futures Historical Prices
vixcentral.com
January 23, 2018
11.975
12.725
13.375
13.875
14.325
14.875
15.175
15.700
15.975
Volatility
Days to Expiration
vixcentral.com

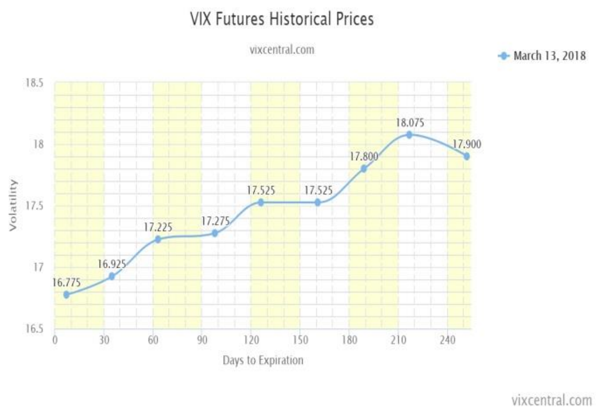

VIX Futures Historical Prices
vixcentral.com
February 2, 2018
15.625
14.975
15.075
15.275
15.425
15.825
15.925
16.225
16.450
Volatility
Days to Expiration
vixcentral.com


VIX Futures Historical Prices
vixcentral.com
February 5, 2018
33.225
27.975
24.725
20.950
19.375
19.425
20.425
18.925
18.975
Volatility
Days to Expiration
vixcentral.com


VIX Futures Historical Prices
vixcentral.com
February 6, 2018
23.875
21.025
20.000
19.225
18.850
18.750
18.150
18.900
18.650
Volatility
Days to Expiration
vixcentral.com

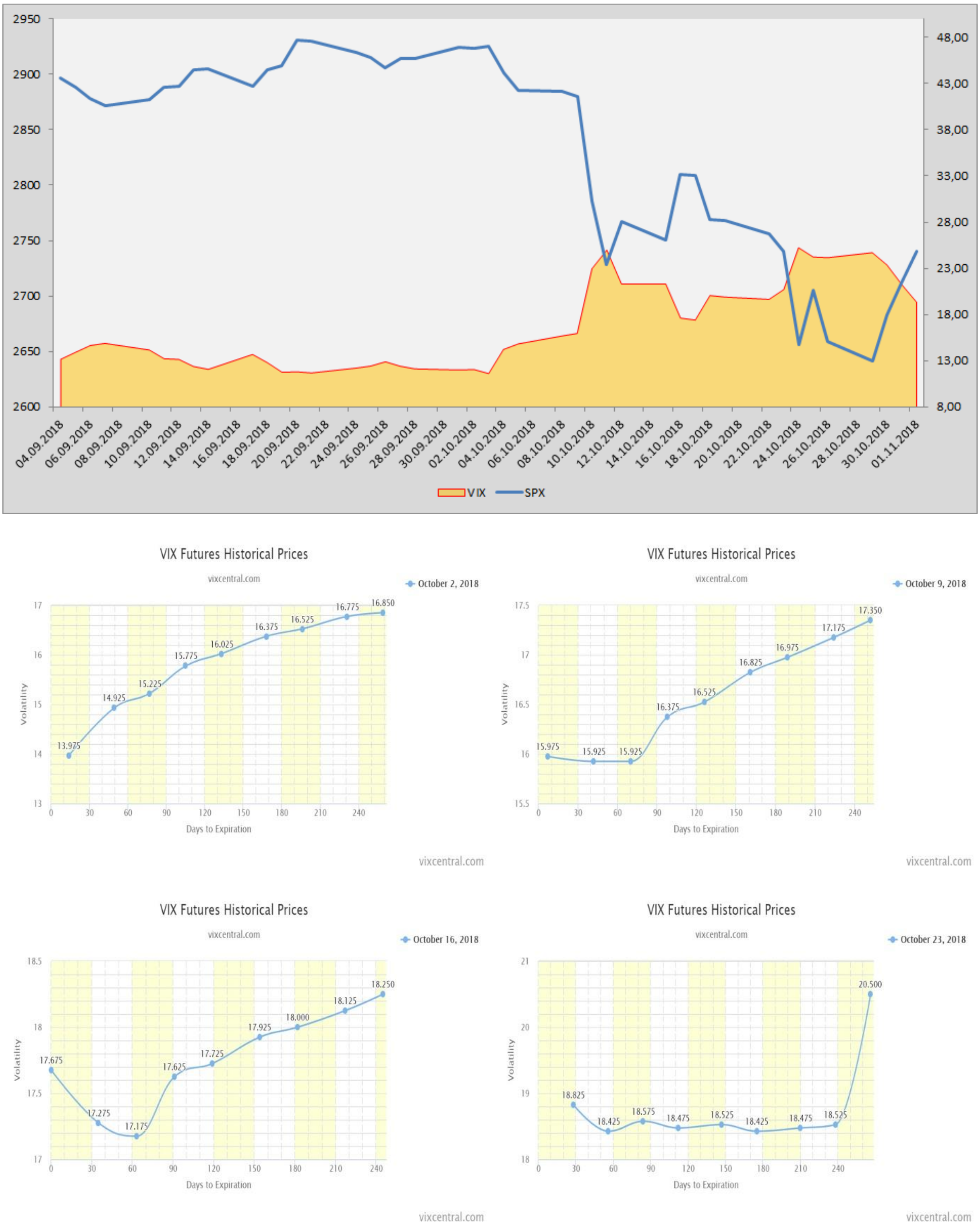

VIX Futures Historical Prices
vixcentral.com
February 13, 2018
25.225
19.825
18.975
18.600
18.175
18.275
18.125
18.325
18.650
Volatility
Days to Expiration
vixcentral.com

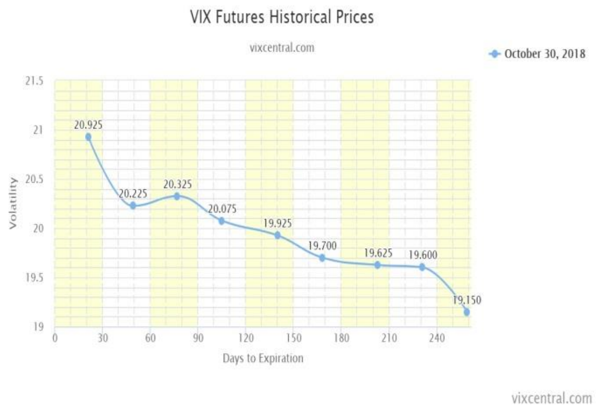

VIX Futures Historical Prices
vixcentral.com
February 14, 2018
17.875
17.775
17.825
17.625
17.825
17.725
17.925
18.075
Volatility
Days to Expiration
vixcentral.com


VIX Futures Historical Prices
vixcentral.com
March 13, 2018
16.775
16.925
17.225
17.275
17.525
17.525
17.800
18.075
17.900
Volatility
Days to Expiration
vixcentral.com

Similarly, Figure 2.3 illustrates the evolution of the curve during the start of the major down-move in SPX in October 2018. The shape of the curve “breathes” with the moves of the SPX and VIX and provides reasonable signals for entering or exiting SVXY (or general VIX based) positions.

Fig. 2.3:





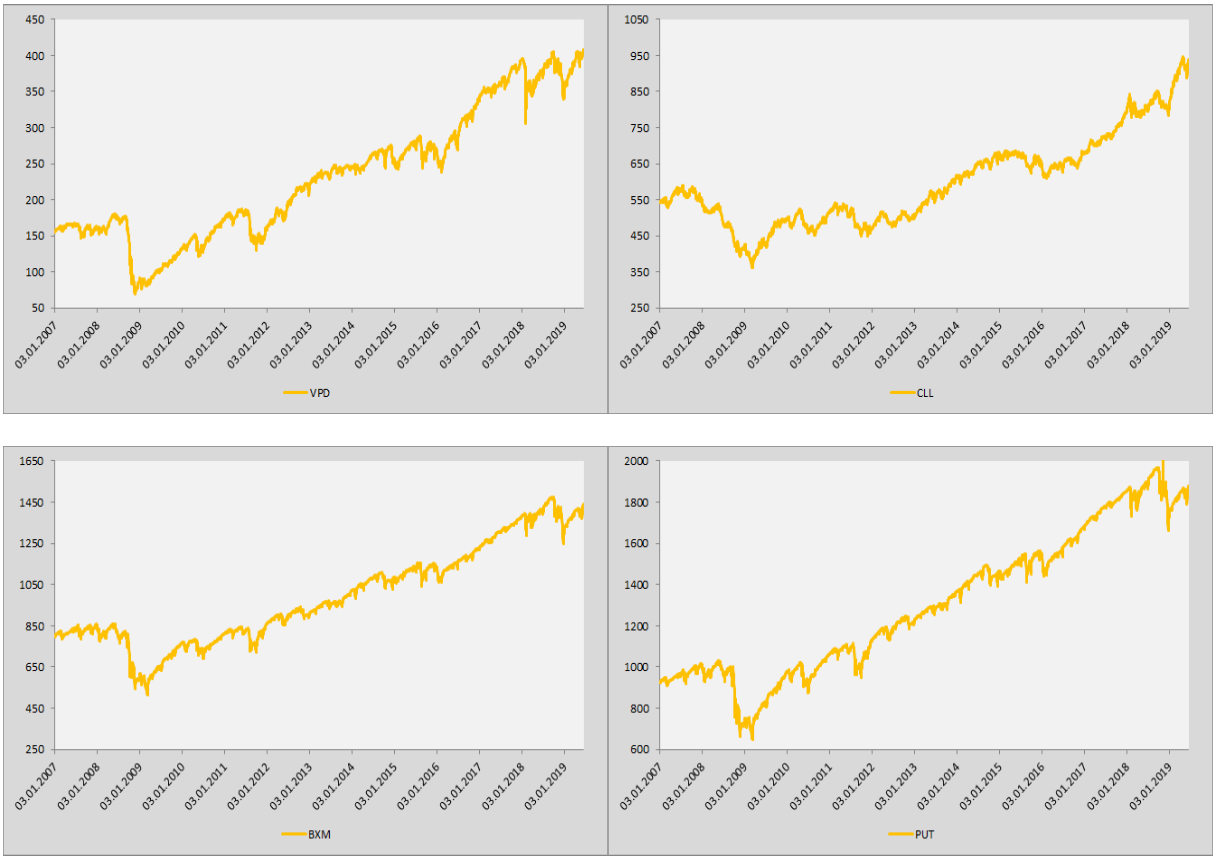

Alternative exit signals for curve-carry positions employed in the market include the difference of VIX to its own moving average of some length (typically 12-20 days), or the VIX level relative to the VXV, the CBOE 3-month volatility index, as an indicator of curve shape. Non-dynamic hedging strategies can be structured by standard Put Options in SVXY or VXX Call options. Both option hedges would suffer from skew (higher implied volatility for rising VIX levels, similar to the SPX skew for Put Options), which can make this hedge prohibitively expensive. Note that hedging SVXY directly with a VXX position would only be a good hedge, if the timing is precise, due to the mismatch in leverage (1 vs. -0.5).

Instead of going long the SVXY to capture the curve effect, one can also go synthetically short the VXX via options, selling VXX Calls and buying VXX Puts. This would be monetizing the skew for VXX Calls as a potential additional premium.

**Curve Premium vs. Option based Time-Value Strategies**

To compare the monetization of the VIX curve premium with alternative time value trades using SPX Options, the charts below illustrate the performance path since 2007 of several CBOE S&P Strategy Indices: The VPD tracks a strategy, which sells the front VIX Futures contract and holds it to expiration, at which time the new front month is shorted. The PUT, which sells ATM SPX Puts, the BXM, which buys the SPX and sells ATM SPX Calls and the CLL, which buys the portfolio of SPX stocks, collects dividends, buys quarterly, 5% OTM Puts on SPX and sells monthly 10% OTM Calls on SPX. The VPD achieves the highest return over the period, but suffers also larger (and very rapid) drawdowns. The CLL has the lowest volatility of these indices and its drawdowns appear at a tempered pace relative to the sudden and rapid drawdowns of the other indices.

Fig. 2.4:




## 2.3 Curve Trading

### Curve Trading with long/short ETPs

The goal of VIX curve trading is to capture various aspects of the general roll effect of Futures along the curve, while at the same time avoiding directional risks. The main instruments are the VIX Futures with expirations from the front month out to about the 7th month. Somewhat simpler but in spirit similar versions are realized by combining ETPs that reflect different parts of the curve (VXX vs. VXZ)

The S&P 500 Dynamic VIX Futures Index (see section 1.6) formalizes a curve strategy, which combines positions in the VXX (Short-Term Futures Index ETN) and the VXZ (Mid-term, 3 month Futures Index ETN), with allocations that change with the steepness of the curve. Specifically, it considers the ratio IVTS (implied volatility term structure) =VIX/VXV as the basis for the allocation weight.

$$DYN = \omega_S VXX + \omega_M VXZ$$

The SPDJ index allocates according to the following Table ("S&P"). There are alternative weighting schemes for the same strategy, e.g., see Donninger (2011) or Sinclair (2013), ("Donn/Sinc").

| S&P | VXX | VXZ |
|---|---|---|
| IVTS < 0,90 | -0,30 | 0,70 |
| 0,90 < IVTS < 1 | -0,20 | 0,80 |
| 1 < IVTS < 1,05 | 0,00 | 1,00 |
| 1,05 < IVTS < 1,15 | 0,25 | 0,75 |
| 1,15 < IVTS | 0,50 | 0,50 |

| Donn/Sinc | VXX | VXZ |
|---|---|---|
| IVTS < 0,91 | -0,60 | 0,40 |
| 0,91 < IVTS < 0,97 | -0,32 | 0,68 |
| 0,97 < IVTS < 1,05 | -0,25 | 0,75 |
| 1,05 < IVTS | -0,10 | 0,90 |

In essence the view expressed by this trade is to be short the VXX, the front ETN, to capture the roll yield of a normally shaped yield curve and hedge potential volatility spikes through a long position in the mid-term ETN, the VXZ, based on the value of the IVTS. The VXZ has an average Delta, which is roughly half of the VXX Delta, which can, however, deviate substantially from that level. The roll yield of the VXX is typically higher than that of the VXZ and hence has a more pronounced drift to the downside in normal market phases.

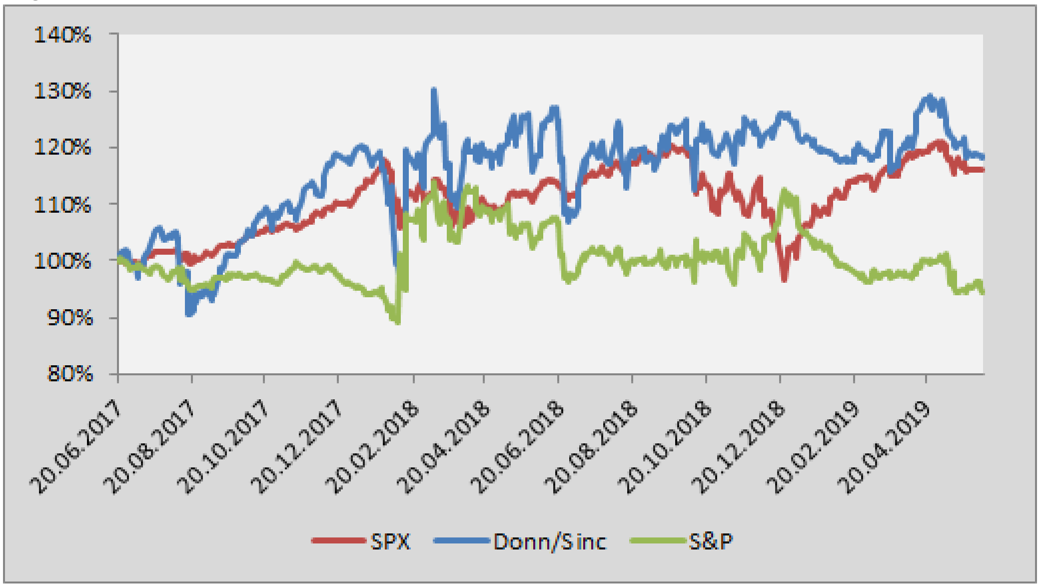
If the IVTS is below 100%, the VXV is higher than the VIX, the longer-term volatilities are higher than short term volatilities which again is indicative of a curve likely in contango. A higher VXV is not to be confused with higher mid-term VIX Futures: The VXV reflects the implied SPX options volatility with a 3 month maturity, a 3 month VIX Futures is the market expectation of the SPX 1-month implied volatility in 3 months' time. The positioning in this case reflects the harvesting of the roll yield by a negative VXX position, while balancing the Delta with the positive VXZ position.

For IVTS levels close to 1, the curve could be in transition from one shape to the other and a more balanced weighting is applied.

If the IVTS is higher than 100%, the VIX is priced at a premium to the mid-term volatilities, so the overall Delta is skewed to the positive and the overall roll yield also profits from a potential backwardation of the VIX curve.

Figure 3.1 shows the performance paths of both realizations of the strategy along with the SPX performance.

Fig. 3.1:



The strategy can also be pursued with SVXY instead of short positions in the VXX, switching back to the VXX whenever its weights become positive. The weighting scheme needs to be derived afresh, as the SVXY has a different participation factor than a short position in the VXX.

In a typical calendar year, 85% to 95% of the time, the VXV exceeds the VIX. Even in in 2008 on 53% of the trading days the VXV exceeded the VIX, so the number of necessary adjustments to the ETP weightings is rather low.

**Curve Trading via Calendar Spreads**

Calendar spreads combine two opposing positions at two points of the VIX curve to benefit from a change in the VIX Futures spread between these two points. It takes a long or short position in an instrument with one expiration and enters the opposite position in an instrument with another expiration. The view expressed is on the direction of the spread between the chosen expirations to widen or to narrow. The instruments used for a calendar spread can be two Futures, two options or a cross combination of a Future with an option.

Calendars spreads in Options on SPX, selling short-term options and buying long-term options, monetize the dominating theta of the short term vs. the longer term option. A SPX calendar spread is typically Gamma short and Vega long and has a dominating short-term delta after sharp moves against the option. All of this may not be the case in corresponding calendar spreads on VIX, as changing curve effects affect the pricing of Options and the Greeks in nontrivial ways. Calendar spreads in VIX Options with a view to monetize the potentially higher short-term Theta, are harder to manage than on SPX. To mention only one effect: Implied volatility of VIX options (as well as realized volatility of the underlying VIX Futures) rises with time decaying as the Delta of the underlying Futures increases. For that reason, the Theta of VIX options has a Vega component which dampens Theta (Theta decreases with increasing implied volatility).

In what follows, the focus will be only on Option-calendar spreads on VIX with a view to monetize the curve premium rather than Theta monetization.

Given that contango is the dominant prevailing shape of the curve, a common view expressed via a calendar spread is that the spread of a short (or even the front) month and a longer term month will widen, as the shorter expiration rolls down faster than the second month. Which points along the curve are used, depends on the current shape of the curve and the view expressed.

Figure 3.2 illustrates the path of the “on the run” front month, the “on the run” fourth month as well as their spread (fourth – front) from September 2018 to June 2019.

Fig. 3.2:

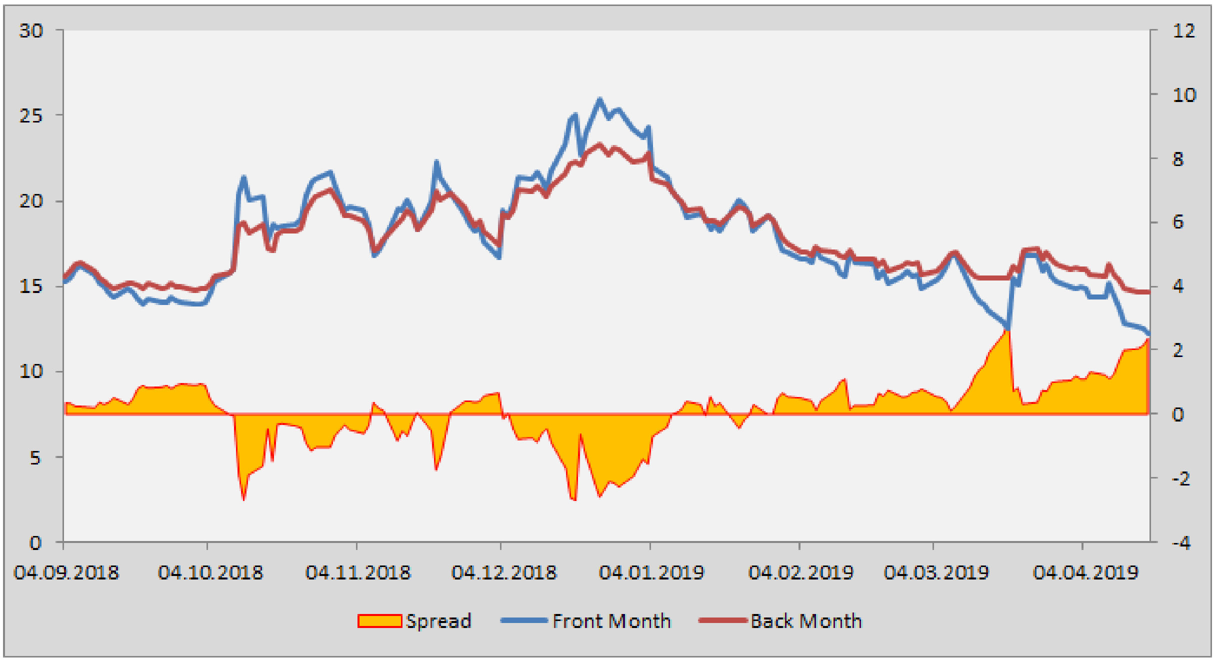
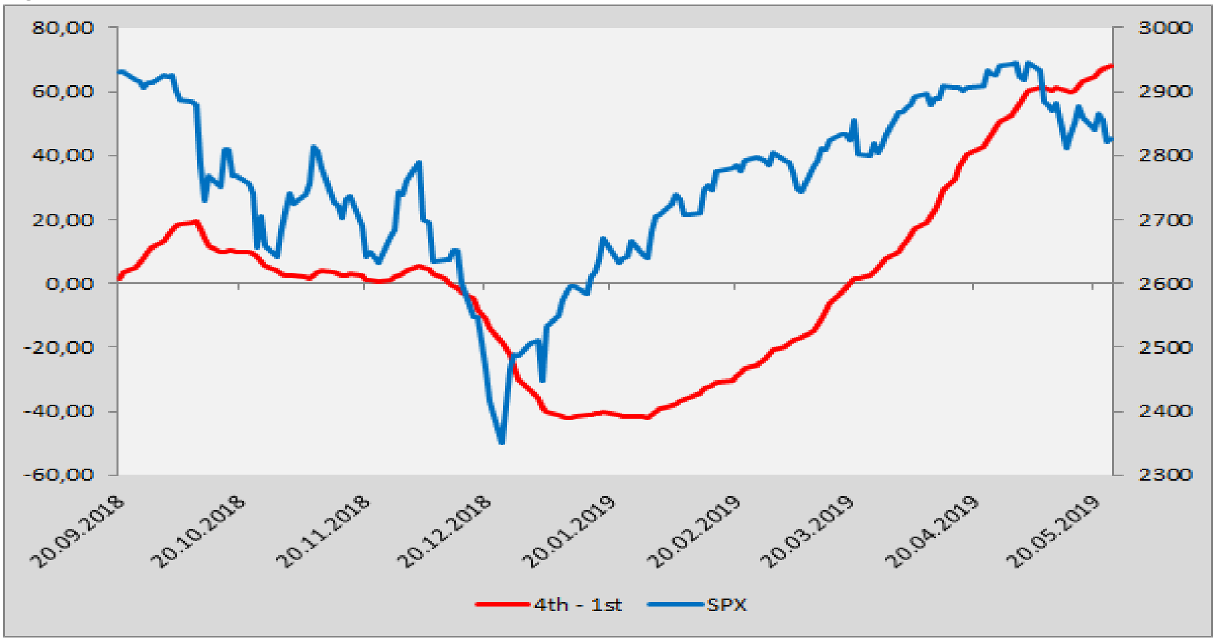



Figure 3.3 illustrates the performance of a position in the spread between fourth and front month (short front, long fourth), which profits if spreads widen (i.e. long the spread) in front of the performance of the SPX.

Fig. 3.3:



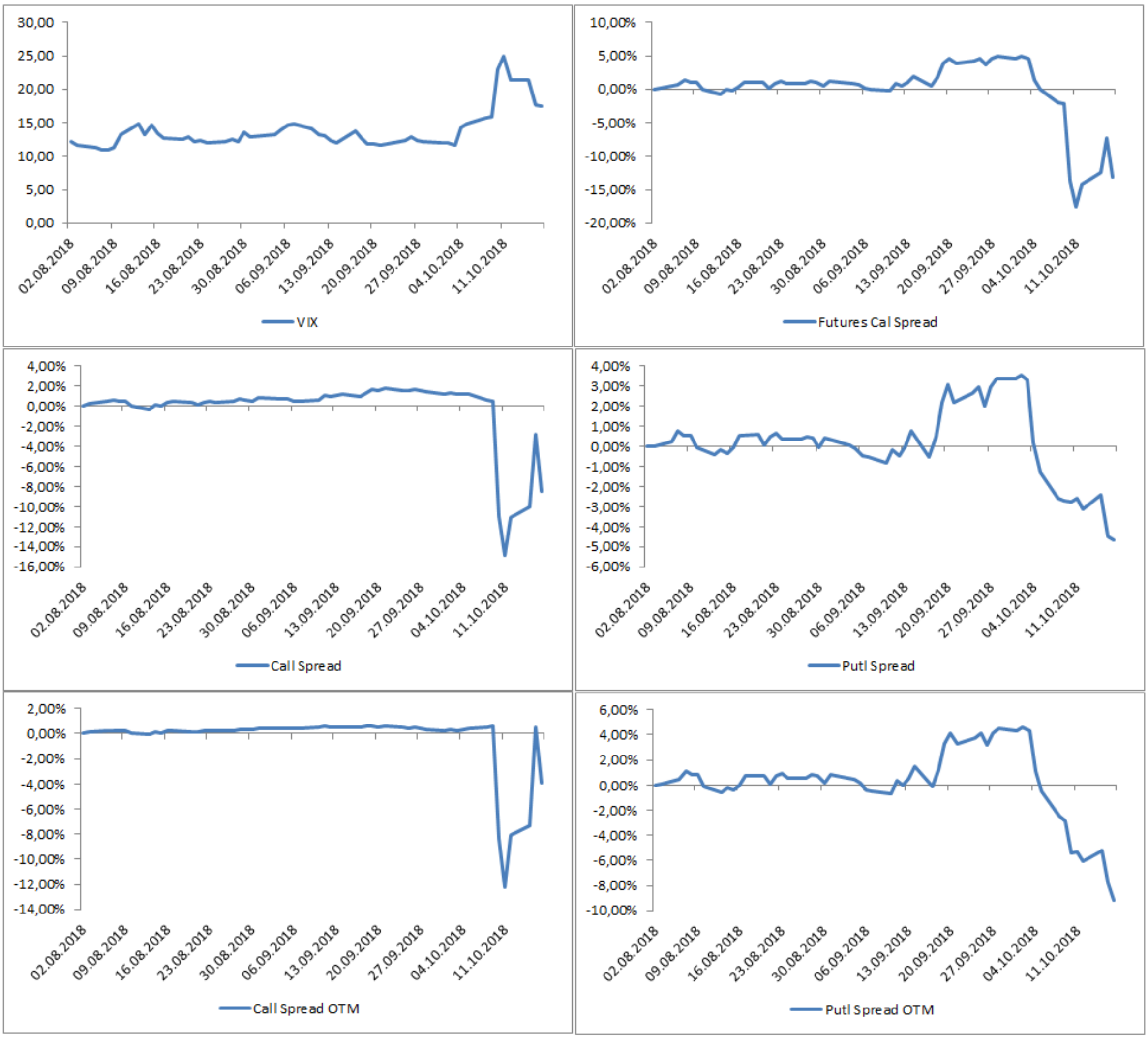

A simple variation to the Futures strategy but avoiding Futures is to use VXX and VXZ, going short VXX against long VXZ, positioning for a higher roll yield of the front two months than the $4^{th}$ – $7^{th}$ month contracts.

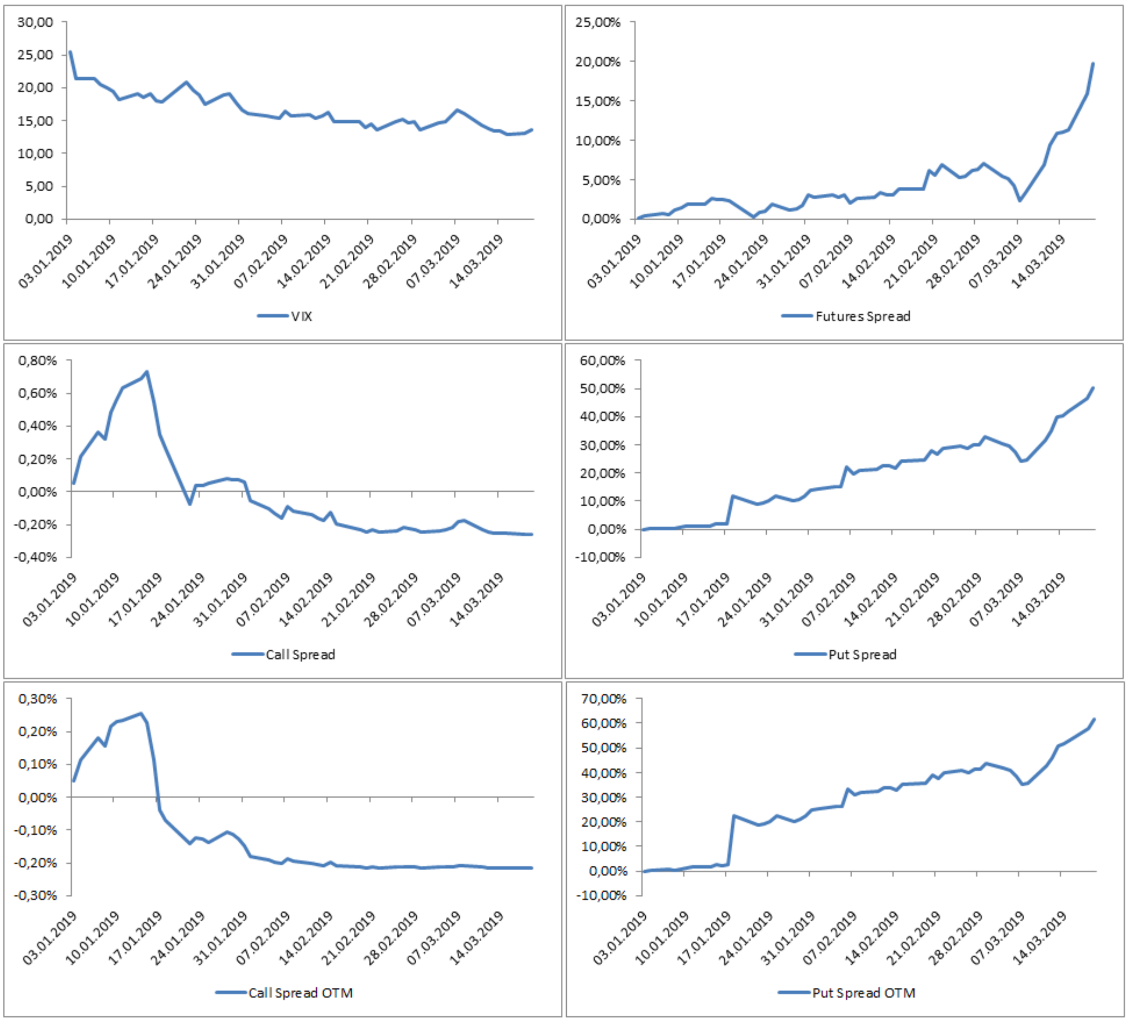
A calendar spread with options would use, e.g., Put options on the VIX. To express the same view as before with Futures, a Put on the shorter expiration will be bought and a Put with longer expiration sold. The strikes are typically not the same, as there may be substantial differences in the value of the underling VIX Futures for the two expirations. However, considerations about expected curve effects will help determine the strikes.

The same view on the curve can also be expressed with Calls and works accordingly, selling the short-term Call against buying the longer-term call. Given that front months price the highest skew, this is another benefit for the Call calendar spread, which sells the highest skew. Put options are priced with lower skew.

Figure 3.4 show the performance path of calendar spreads positioned to benefit from a spread widening, expressed with Futures, Puts and Calls. The performance for pure Option calendar spreads varies with different choices for strikes.

Fig. 3.4:






And finally, the same view on spreads as before can be expressed by combining a Future with an Option position. Generally, combing a Futures position with a long option position rarely turns out well, as the implied volatility on VIX is rather high. Given the rich Option premium, the Options need to move well into the money for the calendar spread to perform well, i.e. the VIX must move sharply. This will be the case during VIX spikes or after a turbulent phase during the reversal to the mean. However, expressing the view of widening spreads in a contango situation would have the long Call position in the combination with Futures (short Futures in front expiration) in the “wrong” expiration: The long Call would be on the long expiration, which reacts more slowly than the short end during VIX spikes, s.th. the Call will move into the money, but not compensate the loss on the front Future. A long Put in combination with Futures would be in the short expiration, which might work out after a VIX spike, in which the mean reversion will pull back the front end faster than the long end.

To monetize the higher implied volatility of VIX Calls, a long Futures position for the back end could be combined with selling a Call for the front maturity. However, this spread trade will only be profitable, when the earned Call premium exceeds any loss of the long Futures position, that is, the back end should be close to its mean and not drop too much, the front should not rise too much to offset the Call Premium – basically a scenario for longer phases of calm markets. Given the higher Delta of the front curve, this position would bear higher VIX-spike risk than a pure Futures calendar spread.

To combine Futures with a sold Put position, the Futures would be short the front, the Puts would be sold for the longer expiration. The premium earned will be lower than in the above case with sold Calls, as the Options expiration is longer, so the implied Volatility as well as the skew for the Put will be lower. The ideal scenario is again one of calm markets and a stable or slightly downward moving curve. The position would again bear a higher VIX spike risk, than a pure Futures spread.

### 2.4 The VIX Curve as a Trading- and Risk Management Signal

The shape of the curve is frequently used to signal potential equity market downturns. Moreover, the level of the VIX as well as the shape of the curve is an indication of risk-sensitivity of market participants. Major market downturns typically force the VIX curve to bend from a normal shape to an inverted curve, starting with the front end, pulling the longer expirees along with it. A curve in contango is the trademark of calm markets, a curve in backwardation typically signals a more turbulent market environment. The graded switching phase from contango to backwardation typically accompanies the transition from calmness to downturn or turbulence. For that reason, changes in the shape of the VIX curve are frequently used to signal entries or exits of market positions in a number of risk-assets, not only SPX. A simple measure of the shape is the IVTS, introduced in section II.3, where it acted as signal to determine position weights.

Already the simple difference, VIX –VXV (Figure 4.1, along with SPX), qualifies as a reasonable signal against fast downward spikes. If a long position in the SPX is exited if this difference becomes positive and re-entered if the difference falls back below zero, it prevents the full participation in several "crash-like" plunges in the SPX (see Figure 4.2). However, it does not prevent more slowly but relentless selling. More elaborate signaling, which might also deliver a warning for general down moves or a Long-Short Strategy in the SPX can be derived by combining the shape of the curve and the recent evolution of it. As such, the VIX curve can be used as a timing signal not only for SPX but also for SPX Option strategies and VIX ETPs.

Fig. 4.1:

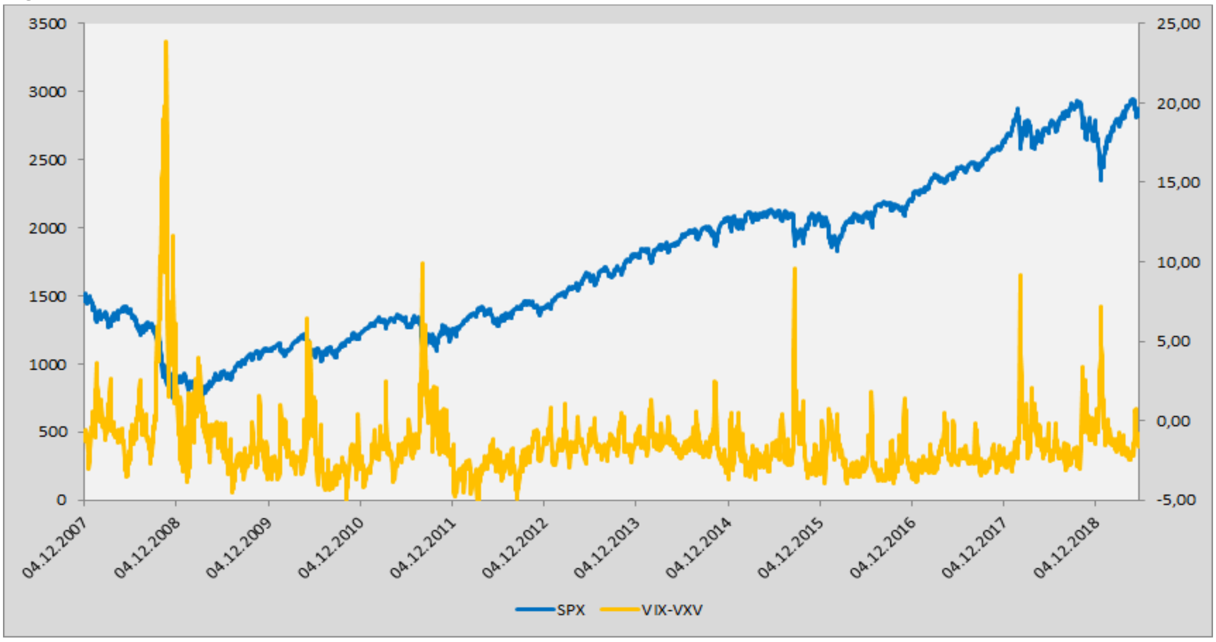
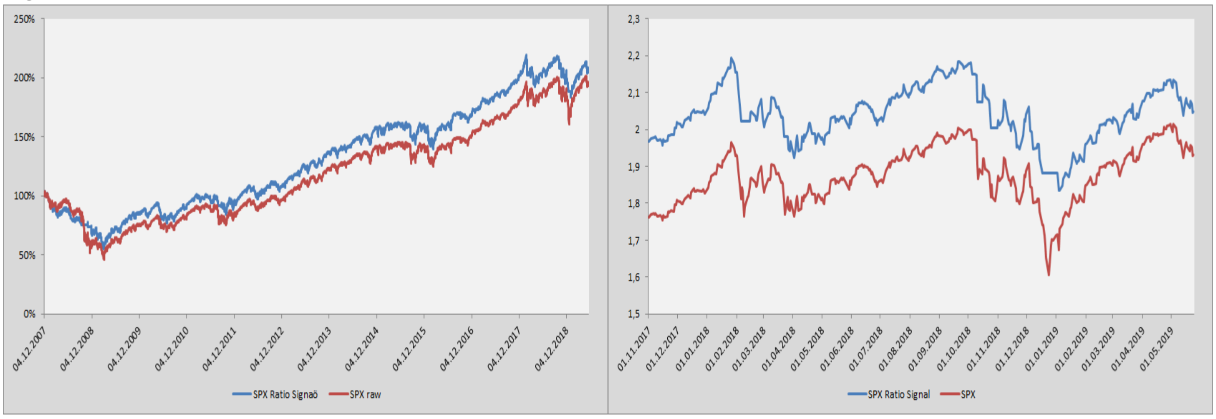


Fig. 4.2:




## 2.5 **Monetizing Imbalances**

Several Imbalances affecting some of the VIX instruments, caused by, e.g., hedging activity or effects relating to differences between implied and realized Volatility, can be monetized. A list would include:

- The VXX related selling of the front VIX Futures (which is continuously rolling out of the first Futures contract to keep a constant average life) creates an imbalance. Due to the large volume traded in VIX ETPs this has become tradable. This effect can so far not be observed for VStoxx, as the volume on related ETPs is currently too small. To monetize this, go long the VIX front month and short the VStoxx front month and potentially reverse the position at the back end to hedge out non parallel shifts of the two curves.
- If the implied volatility of VIX is less than the implied volatility of realized SPX volatility, Options on variance swaps will trade at a premium relative to Options on VIX. Buy options on VIX, sell options on variance swaps.
- Implied volatility for Calls on the VXX will typically trade at a premium to implied volatility on Puts on VXX. This can be monetized by going synthetically short via options (sell call, buy put) against a physical investment in the VXX, or as an alternative to going long the SVXY (Note: SVXY is not a perfect hedge for the VXX).

## **2.6 Option Strategies**

The various effects within the VIX universe from dynamic Futures-Deltas to changing roll yields provide ample and complex playgrounds for Option traders. First, all the classical Option Strategies used for other underlying, can also be employed using VIX or VIX-ETP Options, always keeping the effects and peculiarities of this underlying universe in mind. There is plenty of writing on classical Option based strategies and hence no need to repeat this here, only some of the areas that look promising:

- Implied VIX Volatility provides a similar skew as SPX Put Options, i.e. the Call-Implied Volatilities on long-VIX instruments typically trade higher and giving rise to Collar based strategies monetizing this effect, e.g., capturing roll yield via short Collars.

- Often a very similar Volatility premium as for SPX Options exists and suggests monetization strategies to harvest this premium. If the Delta neutral strategy is chosen, the resulting short-Gamma risk is not to be underestimated and harder to control than for SPX Options.
- The sharp moves in the underlying can also produce large swings in the Delta of the Options, giving rise again to occasional large Gammas, which invites Gamma-scalper. Effects like the rising volatility of VIX-Futures with time, which put a damper on the Gamma have to considered, as well as the fairly high implied Volatility, driving up the cost of Gamma.

Besides classical Options strategies, some VIX based underlying with their complex behavior allow for Options strategies not available for other underlying.

**Trading the Greeks: GammaDeus – the Gamma God. The Case UVXY.**

The UVXY is an ETP tracking twice the daily return of the VIX Short Term Futures Index, for which Options are listed on the NASDAQ. The Gamma aficionados amongst option traders will find in UVXY Options ample and complex playgrounds. Options on UVXY depend on a number of different VIX features and the volatilities – both realized and implied – of the two front VIX Futures.

Options on UVXY are derivatives of another derivative, which is a VIX-based strategy. To peel the onion briefly, options on UVXY have the typical option based relations with respect to the underlying, the UVXY. The UVXY again tracks a 2 times daily return on the SPDJ S&P 500 VIX Short-Term Futures Index. Assuming that the VXX tracks that index perfectly, UVXY can be viewed as a tracker of the twice daily return of VXX, which is a strategy that has to be adjusted on a daily basis to maintain a constant daily leverage of 2. The VXX again is a strategy allocating between the first two VIX Futures V1 and V2, which again follow the VIX Index, but with dynamically changing Deltas of less than one.

As a consequence, Options on UVXY depend on the implied volatility of UVXY, which itself depends on implied and - due to the daily adjustments of notional VXX dependency – on realized volatility of VXX. VXX realized and implied Volatility again depends on the volatilities of the two front VIX Futures and as such, has also a time and VIX-level dependent feature. This again influences Theta and Gamma of all the Options going back up this chain.

Options on the UVXY can be hedged with UVXY, VXX, V1 and V2. In addition, non-linearities or the Greeks can be hedged or traded against via Options on VXX, SVXY and the VIX-Futures.

The standard understanding of the Greeks is only of limited use as the parameters have complex dependencies on the factors mentioned above. Already modeling the Deltas of the VIX Futures depends strongly on the VIX level in a non-robust way and would underlie a number of ad hoc assumptions on the functional relationship if modeled by closed formula.

For the purpose of scenario analysis and a deeper understanding of the way the VIX index as the ultimate underlying affects the Options, the Delta of a UVXY Option with respect to the VIX index can be calculated:

$$\Delta_{\pi,VIX} := \frac{\partial \pi}{\partial VIX} = \frac{\partial \pi}{\partial U}\frac{\partial U}{\partial VXX}\left(\frac{\partial VXX}{\partial V1}\frac{\partial V1}{\partial VIX} + \frac{\partial VXX}{\partial V2}\frac{\partial V2}{\partial VIX}\right)$$

$$= \Delta_{\pi}\Delta_{U,VXX}\left(\Delta_{VXX,V1}\Delta_{V1} + \Delta_{VXX,V2}\Delta_{V2}\right)$$

With $\Delta_{\pi,VIX}$ the Delta of the UVXY-Option with respect to the VIX Index, $\Delta_{\pi}$ the standard Option Delta, $\Delta_{U,VXX}$ the Delta of UVXY with respect to VXX, $\Delta_{VXX,V1,2}$ the Delta of VXX with respect to V1/V2 and $\Delta_{V1,2}$ the Deltas of the two front Futures with respect to the VIX Index.

It is only of theoretical or illustrative purpose, as the VIX Index is no hedging instrument.

For trading purposes, the Delta and Gamma to consider are with respect to the UVXY, VXX or the underlying VIX Futures:

$$\Delta_{\pi,V1,2} := \frac{\partial \pi}{\partial V1,2} = \frac{\partial \pi}{\partial U}\frac{\partial U}{\partial VXX}\frac{\partial VXX}{\partial V1,2}$$

$$= \Delta_{\pi}\Delta_{U,VXX}\Delta_{VXX,V1,2}$$

From section 1.7 we have

$$\Delta_{U,VXX} = \frac{\partial U_t}{\partial VXX_t} = 2\frac{U_t}{VXX_t} \quad and \quad \Delta_{VXX,V1,2} = \frac{\partial VXX_t}{\partial V1,2_t} = \alpha_t, (1-\alpha_t)$$

And hence

$$\Delta_{\pi,V1} = 2\Delta_{\pi}\alpha_t\frac{U_t}{VXX_t} \quad and \quad \Delta_{\pi,V2} = 2\Delta_{\pi}(1-\alpha_t)\frac{U_t}{VXX_t}$$

with $\alpha_t$ the (decaying) position of VXX in the front Future V1.

Consequently, the UVXY Option Delta with respect to the Futures has a dynamic factor on top of the standard Option Delta, which can cause rapid Delta changes and will be reflected in the Gamma with respect to VIX Futures changes. Specifically high realized Volatility of the VIX Index will result in high Volatility of V1, V2 and VXX and produce through the $\frac{U_t}{VXX_t}$ factor a rapidly changing Delta and high Gamma. This effect is partly offset by the dampening influence of high UVXY-Volatility on the standard Gamma. Strong trends in VXX result in a rapidly rising factor and will also lead to higher Gamma, this time supported by the effect of a low realized Volatility of UVXY on the standard Option- Gamma. Only calm markets with VIX near its saturated state will provide for small changes in this factor and result in lower Gamma.

The other Greeks also depend on VXX, V1, V2 and curve effects through their UVXY dependence. What affects all Greeks in various ways are:

- The mean reversion of the VIX index is VIX level dependent, this dependency is passed on to the UVXY Greeks.
- The roll yield provides a bias to VXX and hence UVXY.
- Realized Volatility in the VIX Futures and hence the VXX and UVXY increases with time and is also level dependent. Implied Volatility follows realized Volatility.
- Moneyness changes with the shape of the curve.

In any case, trading the Greeks on UVXY is a challenging exercise. Just focusing only on the standard Greeks with respect to the underlying UVXY would not disclose the full playground of strategies available by using the universe of all VIX instruments that can be traded against UVXY Options.

### 2.7 The Grand Master – setting up a formalized VIX trading operation

All of the previous strategies combine one or several futures or ETPs and change allocation provided by specific rules and/or signals derived from the curve. In essence, they are all special cases of a Grand Master ("GM") strategy, which takes positions in all (or the first 7) Futures contracts with weights $\omega_i$ , options on VIX with weights $\gamma_j$ and expirations $T_j$ and ETPs with weights $\delta_n$ .

The weights can attain negative as well as positive values. They are functions of time, the whole futures curve, the Options chain and potentially other covariates Z (like signals on SPX), whose functional form and input list is determined by learning or back-testing assumptions on historical data.

$$GM_t = \sum_{i,j,n} \omega_i (t) V_i(t) + \gamma_j (t) O_{j,T_j}(t) + \delta_n (t) ETP_n(t)$$

$$\omega_i (t) = F_i[t, V_1(t), \dots, V_M(t), Z(t)]$$

$$\gamma_j (t) = G_j[t, V_1(t), \dots, V_M(t), OC(t), Z(t)]$$

$$\delta_n (t) = H_n[t, V_1(t), \dots, V_M(t), ETP_1(t), \dots, ETP_N(t), Z(t)]$$

If realistic processes for the instruments can be constructed, an optimization scheme could be set up as a stochastic control problem. Alternatively, the optimization problem could be formulated as a learning exercise on past data of a trading policy via Reinforcement Learning.

In a more humble approach for a trading setup, the $F_i$, $G_j$ and $H_n$ could determine first which general strategy (directional, roll yield, curve spread) would be most appropriate in the current market environment. They would set non-applicable weights equal to zero and only use those components taking part in the appropriate strategy. A second step provides the rules for the specific, non-zero weights of the appropriate strategy, which determine the points of the curve to be used, entry and exit signals etc.

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